

Дифференциалдық есептеулер

№ 2 бақылау жұмысы

1. Функциялардың туындысын табу керек.
2. Функцияның локальді экстремумдерін және монотонды аралықтарын табу керек.
3. Функцияның иілу нүктелерін және ойыс-дөңес аралықтарын табу керек.
4. Функция сызбасының асимптоталарын табу керек.

1.

1.

а) $y = x^2 \sin x + \frac{\operatorname{ctgx}}{5(x-1)}$; б) $y = \ln(x^2 + x) + \operatorname{arctg}(2\sqrt{x})$ в) $(\operatorname{tg} x)^x$.

$$2. \quad y = 3\sqrt[3]{x^2} - 4. \quad 3. \quad y = \frac{x}{x^2 - 4}. \quad 4. \quad y = \sqrt[5]{\frac{x}{x-2}}.$$

2

1.

a)

$$y = x^4 \cdot 2^x + \frac{\arcsin x}{3x};$$

б)

$$y = \frac{1 - \sqrt{1 - x^2}}{x^2};$$

$$в) (\cos x)^{\operatorname{tg} x}.$$

$$2. \quad y = \frac{2x^2 - 1}{x^4}. \quad 3. \quad y = \frac{x}{x^3 + 2}. \quad 4. \quad y = \sqrt[3]{x^3 - x^2}.$$

3

1.

$$a) \quad y = 4x^3 \log_2 x + \frac{x}{e^x};$$

$$б) \quad y = (2x + 1) \ln^2 x;$$

$$в) \quad y = \left(\frac{1}{x}\right)^{\ln x}.$$

$$2. \quad y = x - 2 \ln x. \quad 3. \quad y = \frac{x^3 - 3x}{x^2 - 1}. \quad 4. \quad y = \frac{\sqrt{|x^2 - 3|}}{x}.$$

4

1.

$$a) \quad y = \frac{2}{3x-1} - \sqrt{x} \operatorname{tg} x;$$

$$б) \quad y = (\sin^2 x + 4)^2;$$

$$в) \quad y = (\ln x)^{\cos x}.$$

$$2. \quad y = \ln x - 2 \arcsin x. \quad 3. \quad y = \frac{x^4}{x^3 - 1}.$$

$$4. \quad y = 3x + \operatorname{arctg} 5x.$$

5

1.

a)

б)

в)

$$y = \frac{x^2}{1 + \cos x} + x \arcsin x; \quad y = \frac{1}{4} \ln \frac{1+x}{1-x} + \frac{1}{2} \operatorname{arctg} x; \quad y = (\operatorname{ctg} x)^{x^2}.$$

$$2. \quad y = x \cdot e^{-\frac{x}{2}}. \quad 3. \quad y = \frac{x^3}{2(x-1)^2}. \quad 4. \quad y = \frac{\ln(x+1)}{x^2} + 2x.$$

6

1.

a)

$$\text{б) } y = \sqrt[5]{(2+4x^2)^3}; \quad \text{в) }$$

$$y = \frac{x}{2} \arccos x - \frac{5x^2}{3+2x^3}; \quad y = \left(1 + \frac{1}{x}\right)^x.$$

$$2. \quad y = \frac{x}{1+x^3}. \quad 3. \quad y = \frac{(x^2-5)^3}{125}. \quad 4. \quad y = \frac{\sin x}{x}.$$

7

1.

a)

$$\text{б) } y = e^{5x} \cdot \cos^4 \frac{x}{4}; \quad \text{в) } y = (\ln x)^{\sin x}.$$

$$y = \sqrt{x} \cdot 3^x + \frac{\operatorname{arccotg} x}{3x-2};$$

$$2. \quad y = 1 - \sqrt[3]{(x-4)^2}. \quad 3. \quad y = 2^{\frac{1}{x}}. \quad 4. \quad y = x \ln \left(e + \frac{1}{x}\right).$$

8

1.

$$\text{a) } y = 4\sqrt[4]{x^3} \cdot \arccos x + \frac{e^x}{e^x - 1}; \quad \text{б) } y = 3^{\sqrt{\cos 2x}}; \quad \text{в) } y = (\operatorname{ctg} x)^{4x}.$$

$$2. y = x\sqrt{1-x}. \quad 3. y = e^{-2x^2}. \quad 4. y = xe^{\frac{1}{x}}.$$

9

1.

a)

$$y = x^5 \cdot \operatorname{arctg} x - \frac{x-1}{x^2+1};$$

b) $y = \cos^4(\sin \frac{x}{3});$ b) $y = x^{\arccos x}.$

$$2. y = \frac{1+x+x^2}{1-x+x^2}. \quad 3. y = xe^{1-x}. \quad 4. y = \frac{1}{x} e^{-\frac{1}{x}}.$$

10

1.

a)

$$y = 2x^3 \sin x - \frac{3x^2}{x-1};$$

b) $y = \sqrt{\operatorname{arccctg} \frac{x}{2}};$ b) $y = x^{2\operatorname{tg} x}.$

$$2. y = xe^{-\frac{x^2}{2}}. \quad 3. y = \frac{1}{x^4} - \frac{2}{x^2}. \quad 4. y = x \operatorname{arctg} x.$$

11

1.

б)

в)

a) $y = x^4 \arccos x + \frac{\ln x}{\operatorname{ctg} x};$ $y = (3x + \cos 2x)^3 \cdot \operatorname{arctg} 4x;$ $y = x^{\sqrt{x}}.$

$$2. y = e^{2x-x^2}. \quad 3. y = \frac{\ln x}{x}. \quad 4. y = \sqrt[3]{x(x-1)^2}.$$

12

1.

$$\text{a) } y = 2^x \log_2 x + \frac{x^3}{2+e^x}; \quad \text{б) } y = \sqrt{1-e^{-x^2}}; \quad \text{в) } y = (\cos x)^{\arctg x}.$$

$$2. y = \sqrt[3]{x(x-1)^2}. \quad 3. y = \frac{x^3}{x^4-1}. \quad 4. y = \frac{x}{\sqrt{x^2+1}}.$$

13

1.

$$\text{a) } y = \sqrt[3]{x} \cdot \sin x + \frac{3^x}{\arccos x}; \quad \text{б) } y = \arctg \sqrt{x^2-1}; \quad \text{в) } y = \left(\frac{1}{x}\right)^{\text{ctg} x}.$$

$$2. y = x - 2\sqrt{x}. \quad 3. y = x^7 + 7x + 1. \quad 4. y = \frac{x^3}{2(x-1)^2}.$$

14

1.

$$\text{a) } y = \ln x \cdot \sin x + \frac{x - \arctg x}{x^3}; \quad \text{б) } y = \sqrt{1-x^2} \arcsin x; \quad \text{в) } y = (\text{tg} x)^{\frac{1}{x}}.$$

$$2. y = \frac{1-x+x^2}{1+x-x^2}. \quad 3. y = x^4 - 6x^2. \quad 4. y = \frac{2x^3}{x^2-4}.$$

15

1.

$$\text{a) } y = x^4 \arctg x + \frac{\ln x}{\sqrt{x}}; \quad \text{б) } y = 2^{\frac{3x}{\ln x}}; \quad \text{в) } y = (\sqrt{1+x^2})^{\text{tg} x}.$$

$$2. y = \operatorname{arctg} \frac{1-x}{1+x}. \quad 3. y = xe^{2x} + 1. \quad 4. y = 5x + \frac{\ln x}{x}.$$

16

1.

a)

$$y = \sqrt[3]{x^4} \operatorname{tg} x + \frac{\arccos x}{e^x};$$

б)

$$y = 4^{\sqrt{\cos^3 x}};$$

в) $y = (\sin x)^{\operatorname{arctg} x}.$

$$2. y = x\sqrt{1-x^2}. \quad 3. y = \sqrt[3]{(x-4)^5} + 2. \quad 4. y = \sqrt{\frac{x^4}{x^2-1}}.$$

17

1.

a)

$$y = x^4 e^x - \frac{\operatorname{tg} x}{\arcsin x};$$

б) $y = \sqrt[3]{1-\sqrt{x}};$

в) $y = \left(\frac{1}{x}\right)^{x^2}.$

$$2. y = \frac{1}{4} x^2 (x^2 - 3)^2. \quad 3. y = \sqrt[3]{x^2 - 2x}. \quad 4. y = \frac{x^3 - 4x^2 + 3x}{x^2 + 1}.$$

18

1.

a)

$$y = x^2 \sqrt[3]{x} \ln x + \frac{3 \sin x}{1 - \cos x};$$

б)

$$y = x \arcsin \ln x;$$

в) $y = (\sqrt[4]{x})^{\sqrt{x}}.$

$$2. y = \frac{x^2 - 1}{x^2 + 1}. \quad 3. y = \ln(x + \sqrt{x^2 + 1}). \quad 4. y = 3x - \frac{\sin x}{x}.$$

19

1.

a)

$$y = \sin x \cdot \ln x - \frac{\operatorname{arctg} x}{2^x};$$

б)

$$y = \frac{1}{(2\operatorname{tg}^3 4x - 3)^4};$$

в)

$$y = (\sin x)^{\sqrt{x}}.$$

$$2. y = \frac{x}{2-x^3}. \quad 3. y = \frac{1}{\sqrt[3]{x+1}} + \frac{1}{\sqrt[3]{x-1}}. \quad 4. y = \frac{x}{2} + \operatorname{arcc} \operatorname{tg} x.$$

20

1.

$$a) y = x^3 \operatorname{arcc} \operatorname{tg} x - \frac{2\sqrt[3]{x}}{e^x + 1}; \quad б) y = \ln \sqrt[3]{\frac{x^2 - 1}{x^2 + 1}}; \quad в) y = (\operatorname{tg} x)^{2x}.$$

$$2. y = \frac{1}{6} x^3 (x^2 - 5). \quad 3. y = x e^{4x} + 2. \quad 4. y = 2x + \operatorname{arctg} 7x.$$

21

1.

$$a) y = x \ln x - \frac{x^2}{2x-1}; \quad б) y = 2^{\sqrt{3+x^2 \sin x}}; \quad в) y = (\arcsin x)^x.$$

$$2. y = \frac{x^4}{x^3 + 1}. \quad 3. y = x e^{-\frac{x^2}{2}}. \quad 4. y = \frac{x^2}{\sqrt{x^2 + 1}}.$$

22

1.

$$a) y = x^3 \arccos x + \frac{5e^x}{x-1}; \quad б) y = \sqrt{\operatorname{arctg} \sqrt{x}}; \quad в) y = (\ln x)^{\operatorname{ctg} x}.$$

$$2. y = \sqrt{e^{x^2} - 1}. \quad 3. y = 2^{\frac{1}{x}}. \quad 4. y = (x-2)e^{-\frac{1}{x}}.$$

23

1.

$$\text{a)} y = 4\sqrt[4]{x^3} \sin x + \frac{2x^2}{\ln x + 3}; \quad \text{б)} y = \sqrt{4x-1} - \arctg \sqrt{4x-1};$$

$$\text{в)} y = (\arccos x)^{\frac{1}{x}}.$$

$$2. y = 2x - 3\sqrt[3]{x^2}. \quad 3. y = \frac{\ln x}{x}. \quad 4. y = \frac{\sqrt[3]{x^3+2}}{x}.$$

24

1.

$$\text{a)} y = x^2 \sin x + \frac{\operatorname{tg} x}{\sqrt{x}}; \quad \text{б)} y = \ln(e^{-x} + xe^{-x}); \quad \text{в)} y = x^{\arctg x}.$$

$$2. y = \frac{x^2-4}{x^2+4}. \quad 3. y = (x-1)^{\frac{5}{3}}. \quad 4. y = \frac{x^3-2x}{x^2-2}.$$

25

1.

$$\text{a)} y = x \cos x + \frac{\operatorname{tg} x}{2+3\operatorname{ctg} x}; \quad \text{б)} y = \sqrt{\arcsin^2 3x-1}; \quad \text{в)} y = (\ln x)^{\arctg x}.$$

$$2. y = \frac{3x^2+4x+4}{x^2+x+1}. \quad 3. y = (x+2)^5 + 3x-1. \quad 4. y = \frac{4x^4+3x^3+1}{x^3}.$$

26

1.

$$\text{a) } y = \sqrt[3]{x}e^x + \frac{\operatorname{arccctg} x}{1+x^2}; \quad \text{б) } y = \sin^2 \frac{x}{3} \operatorname{ctg} \frac{x}{2}; \quad \text{в) } y = (\sqrt{x})^{\operatorname{arccos} x}.$$

$$2. y = (x^2 - 2x) \ln x - \frac{3}{2}x^2 + 4x. \quad 3. y = x^3 + 2x^2.$$

$$4. y = \sqrt[3]{x^3 + 3x^2}.$$

27

1.

$$\text{a) } y = x^2 \log_5 x - \frac{x}{x^2 + 1}; \quad \text{б) } y = \cos^2 \frac{1 - \sqrt{x}}{1 + \sqrt{x}}; \quad \text{в) } y = (\operatorname{arctg} x)^{x^2}.$$

$$2. y = \sqrt[3]{(x+1)^2} - \sqrt[3]{x^2} + 1. \quad 3. y = x + \frac{\ln x}{x}. \quad 4. y = e^{\frac{1}{x^2 - 4x + 3}}.$$

28

1.

$$\text{a) } y = (x^2 - 2x + 3)e^x + \frac{\operatorname{arcsin} x}{\operatorname{arccos} x}; \quad \text{б) } y = \ln^4 \sqrt{1+x^2}; \quad \text{в) } y = x^{\operatorname{tg} x}.$$

$$2. y = e^{\frac{1}{x^2 - 4x + 3}}. \quad 3. y = \frac{2x-1}{(x-1)^2}. \quad 4. y = \sqrt{\frac{x^3 - 2}{3x}}.$$

29

1.

$$\text{a) } y = -x \operatorname{arctg} x + \frac{x-1}{\log_2 x}; \quad \text{б) } y = e^{\sqrt{\ln^2 x + 3x}}; \quad \text{в) } y = (\sin x)^{\cos x}.$$

$$2. y = \frac{x}{e^x}. \quad 3. y = x - 2\operatorname{arctg} x. \quad 4. y = e^{\frac{1}{x}} - x.$$

30

1.

a)

$$y = x^2 \ln x + \frac{2x}{\sin x + \cos x};$$

б)

$$y = \arcsin \sqrt{\frac{1-x}{1+x}};$$

в)

$$y = (\operatorname{tg} x)^{\operatorname{arctg} x}.$$

$$2. y = -x^2 \sqrt{x^2 + 2}. \quad 3. y = \frac{(x^2 - 1)(x - 2)}{x}. \quad 4. y = \frac{x^3}{3(x - 1)^2}.$$

Интегралдық есептеулер

№3 бақылау жұмысы

Интегралдарды табу керек.

1

$$1. \int \left(\frac{1-x}{x}\right)^2 dx; \quad 2. \int (e^x + 1)^3 dx; \quad 3. \int \arccos 2x dx; \quad 4. \int_1^8 \frac{dx}{\sqrt[3]{x}}.$$

2

$$1. \int \frac{(1-x)^2}{x\sqrt{x}} dx; \quad 2. \int \frac{dx}{(4x+1)^3}; \quad 3. \int x \cos 5x dx; \quad 4. \int_1^e \frac{\cos \ln x}{x} dx.$$

3

$$1. \int (\sqrt{x} + 1)(x - \sqrt{x}) dx; \quad 2. \int \frac{e^x dx}{e^x + 5}; \quad 3. \int x^3 \ln x dx;$$

$$4. \int_1^6 \frac{dx}{1 + \sqrt{3x - 2}}.$$

4

$$1. \int \frac{\sqrt[3]{x^2} - \sqrt[4]{x}}{\sqrt{x}} dx; \quad 2. \int \frac{2x dx}{x^4 + 1}; \quad 3. \int \frac{\ln x dx}{\sqrt{x}}; \quad 4. \int_{\ln 3}^{\ln 8} \frac{e^x dx}{\sqrt{e^x + 1}}.$$

5

$$1. \int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} dx; \quad 2. \int \frac{dx}{\arcsin^2 x \sqrt{1 - x^2}};$$

$$3. \int (x^2 - x + 1) \ln x dx; \quad 4. \int_0^{\frac{\pi}{4}} e^{3x} \sin 4x dx.$$

6

$$1. \int \frac{1 + \cos^2 x}{1 + \cos 2x} dx; \quad 2. \int \frac{\arctg^3 x dx}{1 + x^2}; \quad 3. \int x^2 \sin 3x dx; \quad 4. \int_0^{\frac{\pi}{2}} \cos^3 x dx.$$

7

$$1. \int 2 \cos^2 \frac{x}{2} dx; \quad 2. \int \frac{\sqrt[3]{\ln x}}{x} dx; \quad 3. \int x^2 e^{-2x} dx; \quad 4. \int_0^1 \frac{x^2 dx}{1 + x^6}.$$

8

$$1. \int \frac{(1 + 2x^2) dx}{x^2(1 + x^2)}; \quad 2. \int x \sqrt{4 - x^2} dx; \quad 3. \int x^3 e^x dx; \quad 4. \int_0^1 \frac{dx}{4x^2 + 4x + 5}.$$

9

$$1. \int \frac{(1 + x)^2}{x(1 + x^2)} dx; \quad 2. \int \sqrt{4 - 3x} dx; \quad 3. \int x^3 e^{-x^2} dx; \quad 4. \int_0^{\pi} \sin^3 x dx.$$

10

$$1. \int \frac{dx}{\cos 2x + \sin^2 x} dx; \quad 2. \int \frac{(2x - 3) dx}{2\sqrt{x^2 - 3x + 1}}; \quad 3. \int \frac{\ln^2 x}{x^2} dx; \quad 4. \int_0^2 \frac{2x - 1}{2x + 1} dx.$$

11

$$1. \int \frac{x-2}{x} dx; \quad 2. \int \frac{x^3 dx}{\sqrt{x-1}}; \quad 3. \int x 2^x dx; \quad 4. \int_1^2 x \log_2 x dx.$$

12

$$1. \int \frac{1+x^3}{x+1} dx; \quad 2. \int \frac{(2x-5)dx}{x^2-5x+3}; \quad 3. \int \arctg \sqrt{x} dx; \quad 4. \int_0^\pi x \sin x dx.$$

13

$$1. \int \left(\frac{1+2x}{x}\right)^2 dx; \quad 2. \int \frac{x^3 dx}{\sqrt[3]{x^4+1}}; \quad 3. \int x \arctg x dx; \quad 5. \int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}}.$$

14

$$1. \int \frac{x^2-1}{x^2+1} dx; \quad 2. \int e^{-x^3} x^2 dx; \quad 3. \int \sin(\ln x) dx; \quad 4. \int_0^1 x e^{-x} dx.$$

15

$$1. \int \frac{x^4 dx}{x^2+1}; \quad 2. \int \frac{2^x dx}{\sqrt{1-4^x}}; \quad 3. \int x \cos^2 x dx; \quad 4. \int_0^1 \frac{\sqrt{x} dx}{1+x}.$$

16

$$1. \int \frac{x^3+2}{x} dx; \quad 2. \int \frac{2x-\sqrt{\arcsin x}}{\sqrt{1-x^2}} dx; \quad 3. \int x \sin x \cos x dx;$$

$$4. \int_2^{10} \sqrt{x-2} dx.$$

17

$$1. \int 3^x (1+2x^3 \cdot 3^{-x}) dx; \quad 2. \int \frac{\cos^3 x dx}{\sin^4 x}; \quad 3. \int x \ln(1+x) dx; \quad 4. \int_1^2 \frac{e^{1/x^2}}{x^3} dx.$$

18

1. $\int \frac{2 - \cos^3 x}{\cos^2 x} dx$; 2. $\int \frac{x(1-x^2)}{1+x^4} dx$; 3. $\int x^4 \ln x dx$;
 4. $\int_0^1 \frac{dx}{4x^2 + 4x + 5}$.

19

1. $\int \frac{dx}{x^2 + 9}$; 2. $\int \frac{xe^{\sqrt{x^2-1}}}{\sqrt{x^2-1}} dx$; 3. $\int x^2 \sin x dx$; 4. $\int_1^e \frac{dx}{x(1+\ln^2 x)} dx$.

20

1. $\int \frac{(1+\sqrt{x})^2 dx}{\sqrt{x}}$; 2. $\int \frac{e^x}{(2-e^x)^3} dx$; 3. $\int \frac{\ln^2 x}{x^2} dx$; 4. $\int_0^{\pi/2} \cos^2 x dx$.

21

1. $\int (\sin \frac{x}{2} + \cos \frac{x}{2})^2 dx$; 2. $\int \frac{\arctg^2 x}{1+x^2} dx$; 3. $\int x \cos 3x dx$;
 4. $\int_{\ln 3}^{\ln 8} \frac{dx}{\sqrt{e^x + 1}}$.

22

1. $\int \frac{x^2}{x^2-9} dx$; 2. $\int x(5x-1)^{19} dx$; 3. $\int x^2 \cdot e^{2x} dx$; 4. $\int_0^1 x \arctg x dx$.

23

1. $\int (\frac{1}{\sqrt[3]{x^2}} - \frac{x+1}{\sqrt[4]{x^3}}) dx$; 2. $\int \frac{e^{2x}}{e^x + 1} dx$; 3. $\int (1-x) \ln x dx$;
 4. $\int_0^1 \frac{\arcsin x}{\sqrt{1+x}} dx$.

24

$$1. \int \operatorname{tg}^2 x dx; \quad 2. \int \frac{dx}{x\sqrt{1-4\ln^2 x}}; \quad 3. \int e^{\sqrt{x}} dx; \quad 4. \int_{\pi/6}^{\pi/3} \frac{x dx}{\cos^2 x}.$$

25

$$1. \int \frac{x^2-1}{x+1} dx; \quad 2. \int \frac{dx}{\cos^2 x \operatorname{tg}^4 x}; \quad 3. \int x^2 \sin 2x dx; \quad 4. \int_0^1 \frac{x^2 dx}{1+x^6}.$$

26

$$1. \int (3x + \sqrt{x} - \sin x) dx; \quad 2. \int \frac{(2 + \ln x)^2}{x} dx; \quad 3. \int e^{2x} \cos x dx;$$

$$4. \int_0^1 \frac{x dx}{x^2 + 1}.$$

27

$$1. \int \sin^2 \frac{x}{2} dx; \quad 2. \int \frac{x dx}{\sqrt[3]{x^2-9}}; \quad 3. \int (x^2 + 2x + 1) \cdot e^x dx;$$

$$4. \int_0^1 \frac{\operatorname{arctg} x}{1+x^2} dx.$$

28

$$1. \int \frac{x^2+2}{x^2+1} dx; \quad 2. \int \frac{x^2 dx}{1+x^3}; \quad 3. \int x^2 \cdot \sin 2x dx; \quad 4. \int_0^1 \frac{dx}{e^x+1}.$$

29

$$1. \int e^{2x} (e^{-2x} + e^{-x}) dx; \quad 2. \int \frac{\arccos x dx}{\sqrt{1-x^2}}; \quad 3. \int x^2 \cdot e^{3x} dx; \quad 4. \int_1^e \ln^2 x dx.$$

30

$$1. \int (5 \cos x + x^3 - \frac{1}{x}) dx; \quad 2. \int \frac{x^3 dx}{1+x^8}; \quad 3. \int x \cos 2x dx; \quad 4. \int_0^{\pi/4} \sin 2x dx.$$