

Бақылау жұмысына арналған есептер

№1 бақылау жұмысы.

1. Функцияның шегін табу керек.

2. $y = f(x)$ функциясы және x аргументтің екі мәні берілген.

1) Функцияны x -тің осы мәндерінде үзіліссіздікке зерттеу керек;

2) Функцияның x_1 мен x_2 нүктелерінің маңайындағы схемалық сызбасын салу керек.

3. Құрақталып берілген $y = f(x)$ функциясының үзіліс нүктелерін табу керек және схемалық сызбасын салу керек.

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1. а) $\lim_{x \rightarrow \infty} \frac{9x^5 - 4x^4 + 2}{3x^5 - 2x - 1};$

б) $\lim_{x \rightarrow 2} \frac{2x^2 - 11x + 5}{x^2 - 7x + 10}; \quad \lim_{x \rightarrow 5} \frac{2x^2 - 11x + 5}{x^2 - 7x + 10};$

в) $\lim_{x \rightarrow 1} \frac{\sqrt{5x+4}-3}{\sqrt{2x-1}-1}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\arctg 3x}{4x}; \quad \text{д) } \lim_{x \rightarrow -\pi} \frac{\sin 2x}{x(\pi+x)};$

е) $\lim_{x \rightarrow -2} (5+2x)^{\frac{3}{x+2}}; \quad \lim_{x \rightarrow 0} (5+2x)^{\frac{3}{x+2}}.$

2. $y = e^{\frac{1}{x-7}}, x_1 = 7, x_2 = 0.$

$$3. y = \begin{cases} x+4, & x < -1, \\ x^2+2, & -1 \leq x < 1, \\ 2x, & x \geq 1. \end{cases}$$

$$1. \text{ а) } \lim_{x \rightarrow \infty} \frac{7x^3 - 4x^2 + 6}{3x^3 + 10x^2 + 5x};$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{2x^2 - 13x - 7}{x^2 - 9x + 14}; \quad \lim_{x \rightarrow 7} \frac{2x^2 - 13x - 7}{x^2 - 9x + 14};$$

$$\text{в) } \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{4x+1}-3}; \quad \text{г) } \lim_{x \rightarrow 0} 2x \cdot \operatorname{ctg} 5x; \quad \text{д) } \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\pi}{2} - x \right) \operatorname{tg} x;$$

$$\text{е) } \lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x^2 + 4x - 1} \right)^{-3x^2}; \quad \lim_{x \rightarrow 1} \left(\frac{x^2 + x + 1}{x^2 + 4x - 1} \right)^{-3x^2}.$$

$$2. \quad y = \ln(x-8), \quad x_1 = 7, \quad x_2 = 8.$$

$$3. \quad y = \begin{cases} \cos x, & x < 0, \\ 1-x, & 0 \leq x \leq 2, \\ x^2, & x > 2. \end{cases}$$

$$1. \text{ а) } \lim_{x \rightarrow \infty} \frac{x^4 - 3x^2 + 2}{5x^4 - 3x - 2}.$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{3x^2 - 40x + 128}{x - 8}; \quad \lim_{x \rightarrow 8} \frac{3x^2 - 40x + 128}{x - 8};$$

$$\text{в) } \lim_{x \rightarrow 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1-\cos 3x}{4x^2}; \quad \text{д) } \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x};$$

$$\text{e) } \lim_{x \rightarrow \infty} \left(\frac{10x-3}{10x+1} \right)^{5x}; \quad \lim_{x \rightarrow 1} \left(\frac{10x-3}{10x+1} \right)^{5x}.$$

$$2. \quad y = e^{\frac{1}{x-2}}, \quad x_1 = 2, \quad x_2 = 1.$$

$$3. \quad y = \begin{cases} -x, & x \leq 0, \\ -(x-1)^2, & 0 < x < 2, \\ x-3, & x \geq 2. \end{cases}$$

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$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 + 6}{2x^3 + 10x^2 + 5x};$$

$$6) \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{3x^2 + x - 4}; \quad \lim_{x \rightarrow -\frac{4}{3}} \frac{2x^2 - x - 1}{3x^2 + x - 4};$$

$$\text{b) } \lim_{x \rightarrow 8} \frac{\sqrt{1+x} - 3}{2 - \sqrt[3]{x}}; \quad \text{r) } \lim_{x \rightarrow 0} \frac{\arcsin 5x}{3x}; \quad \text{д) } \lim_{x \rightarrow \pi} \frac{\sin x}{\pi^2 - x^2};$$

$$\text{e) } \lim_{x \rightarrow 3} (4x-11)^{\frac{5x}{x-3}}; \quad \lim_{x \rightarrow 4} (4x-11)^{\frac{5x}{x-3}}.$$

$$2. \quad y = \frac{x-4}{x^2 + x - 20}, \quad x_1 = 4, \quad x_2 = -5.$$

$$3. \quad y = \begin{cases} \sin x, & x < 0, \\ x^2, & 0 \leq x \leq 2, \\ 0, & x > 2. \end{cases}$$

$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{x^3 - 3x^2 + 2}{5x^4 + 8x - 6};$$

$$6) \lim_{x \rightarrow 0} \frac{5x^2 - 51x + 10}{x - 10}; \quad \lim_{x \rightarrow 10} \frac{5x^2 - 51x + 10}{x - 10};$$

$$b) \lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x^2 + x}; \quad r) \lim_{x \rightarrow 0} \frac{3x^2 - 5x}{\sin 3x}; \quad d) \lim_{x \rightarrow 1} \frac{\cos \frac{\pi x}{2}}{1 - \sqrt{x}};$$

$$e) \lim_{x \rightarrow -1} (2x + 3)^{\frac{1}{x+1}}; \quad \lim_{x \rightarrow 0} (2x + 3)^{\frac{1}{x+1}}.$$

$$2. \quad y = 5^{\frac{1}{11-x}}, \quad x_1 = 11, \quad x_2 = 3.$$

$$3. \quad y = \begin{cases} x + 2, & x \leq -1, \\ x^2 + 1, & -1 < x \leq 1, \\ -x + 3, & x > 1. \end{cases}$$

$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{6x^5 - 3x^2 + 1}{3x^5 - 2x + 3};$$

$$6) \lim_{x \rightarrow 2} \frac{2x^2 - 5x - 3}{x^2 - 5x + 6}; \quad \lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 5x + 6};$$

$$b) \lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{\sqrt{2x-1} - 3}; \quad r) \lim_{x \rightarrow 0} \frac{\arctg 8x}{2x}; \quad d) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tg 3x}{\tg x};$$

$$\text{e) } \lim_{x \rightarrow \infty} \left(\frac{3x^2 - 6x + 7}{3x^2 + 20x - 1} \right)^{1-x}; \quad \lim_{x \rightarrow 1} \left(\frac{3x^2 - 6x + 7}{3x^2 + 20x - 1} \right)^{1-x}.$$

$$2. \quad y = \arctg\left(\frac{1}{x+9}\right), \quad x_1 = -9, \quad x_2 = -8.$$

$$3. \quad y = \begin{cases} -x^2, & x \leq 0, \\ \operatorname{tg} x, & 0 < x \leq \frac{\pi}{4}, \\ 2, & x > \frac{\pi}{4}. \end{cases}$$

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$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 + 6}{3x^3 + 10x^2 + 4x};$$

$$\text{б) } \lim_{x \rightarrow 5} \frac{3x^2 - 14x - 5}{x^2 - 6x + 5}; \quad \lim_{x \rightarrow 1} \frac{3x^2 - 14x - 5}{x^2 - 6x + 5};$$

$$\text{в) } \lim_{x \rightarrow -2} \frac{\sqrt[3]{x-6} + 2}{x^3 + 8}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1 - \cos 5x}{x \operatorname{tg} 2x}; \quad \text{д) } \lim_{x \rightarrow \frac{\pi}{4}} \operatorname{tg}\left(\frac{\pi}{4} - x\right) \operatorname{tg} 2x;$$

$$\text{e) } \lim_{x \rightarrow \infty} \left(\frac{13x + 2}{13x - 15} \right)^{x+7}; \quad \lim_{x \rightarrow 1} \left(\frac{13x + 2}{13x - 15} \right)^{x+7}.$$

$$2. \quad y = \ln(x-7), \quad x_1 = 7, \quad x_2 = 5.$$

$$3. \quad y = \begin{cases} x+1, & x \leq 0, \\ (x+1)^2, & 0 < x \leq 2, \\ -x+4, & x > 2. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{x^3 - 4x - 2}{5x^3 + 3x^2 - 1};$

б) $\lim_{x \rightarrow \frac{1}{5}} \frac{2x^2 - 7x + 3}{5x^2 - 16x + 3}; \quad \lim_{x \rightarrow 3} \frac{2x^2 - 7x + 3}{5x^2 - 16x + 3};$

в) $\lim_{x \rightarrow 3} \frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1-\cos 2x}{x \sin x}; \quad \text{д) } \lim_{x \rightarrow 1} \frac{\sin \pi x}{\sin 3\pi x};$

е) $\lim_{x \rightarrow \infty} \left(\frac{5x^2 + 8x - 2}{5x^2 + 3x + 3} \right)^{4x+1}; \quad \lim_{x \rightarrow 1} \left(\frac{5x^2 + 8x - 2}{5x^2 + 3x + 3} \right)^{4x+1}.$

2. $y = x + \frac{x+3}{x^2+3x}, x_1 = -3, x_2 = 0.$

3. $y = \begin{cases} x, & x < 0, \\ x^2 + 1, & 0 \leq x < 1, \\ 3 - x, & x \geq 1. \end{cases}$

1. а) $\lim_{x \rightarrow \infty} \frac{(x+1)^3 + (x-1)^3}{x^3 + 1};$

б) $\lim_{x \rightarrow 0,5} \frac{3x^2 + 5x - 2}{2x^2 + 3x - 2}; \quad \lim_{x \rightarrow -2} \frac{3x^2 + 5x - 2}{2x^2 + 3x - 2};$

в) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1}}{x}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x^3}; \quad \text{д) } \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\pi - 2x};$

$$\text{e) } \lim_{x \rightarrow 1} (3x-2)^{\frac{4x-2}{x-1}}; \lim_{x \rightarrow 1} (2x+3)^{\frac{4x-2}{x+1}}.$$

$$2. \quad y = \arctg \frac{1}{3-x}, \quad x_1 = 3, \quad x_2 = 2.$$

$$3. \quad y = \begin{cases} -\sin x, & x < 0, \\ x^2, & 0 \leq x < 1, \\ 2x, & x \geq 1. \end{cases}$$

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$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{3x^4 - 2}{\sqrt{x^8 + 3x + 4}};$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{x^3 - 8}{2x - 4}; \quad \lim_{x \rightarrow 0} \frac{x^3 - 8}{2x - 4};$$

$$\text{в) } \lim_{x \rightarrow -1} \frac{\sqrt{2x+3}-1}{\sqrt{5+x}-2}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\arctg 5x}{\arcsin 4x}; \quad \text{д) } \lim_{x \rightarrow -2} \frac{\tg \pi x}{x+2};$$

$$\text{e) } \lim_{x \rightarrow \infty} \left(\frac{2x-1}{5x+4} \right)^{x/2}; \quad \lim_{x \rightarrow 1} \left(\frac{2x-1}{5x+4} \right)^{x/2}.$$

$$2. \quad y = e^{\frac{1}{x+5}}, \quad x_1 = -5, \quad x_2 = 1.$$

$$3. \quad y = \begin{cases} x+1, & x < -2, \\ -x^2+2, & -2 \leq x < 1, \\ 2+x, & x \geq 1. \end{cases}$$

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$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{(3x-4)(x+1)}{x^3 + x^2 + 2};$$

$$6) \lim_{x \rightarrow 3} \frac{4x^2 - 8x + 3}{2x^2 - 7x + 3}; \quad \lim_{x \rightarrow 0,5} \frac{4x^2 - 8x + 3}{2x^2 - 7x + 3};$$

$$\text{b) } \lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 3} + 2x}{x + 1}; \quad \text{r) } \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{x^2}; \quad \text{д) } \lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\pi - x};$$

$$\text{e) } \lim_{x \rightarrow 1} (2 - x)^{\frac{4x+5}{x-1}}; \quad \lim_{x \rightarrow 0} (2 - x)^{\frac{4x+5}{x-1}}.$$

$$2. \quad y = \frac{x+2}{x^2 - 25}, \quad x_1 = 5, \quad x_2 = -5.$$

$$3. \quad y = \begin{cases} x-1, & x < -1, \\ -2, & -1 \leq x < 0, \\ \cos x, & x \geq 0. \end{cases}$$

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$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{(x+2)^2 - (x-2)^2}{x+2};$$

$$6) \lim_{x \rightarrow 5} \frac{5x^2 - 2x - 39}{x^2 - 2x + 15}; \quad \lim_{x \rightarrow 3} \frac{5x^2 - 2x - 39}{x^2 - 2x + 15};$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}; \quad \text{r) } \lim_{x \rightarrow 0} 3x \operatorname{ctg} 9x; \quad \text{д) } \lim_{x \rightarrow 2} (x-2) \operatorname{ctg} \frac{\pi x}{2};$$

$$\text{e) } \lim_{x \rightarrow \infty} \left(\frac{6x-7}{6x+5} \right)^{3x-6}; \quad \lim_{x \rightarrow 2} \left(\frac{6x-7}{6x+5} \right)^{3x-6}.$$

2. $y = \ln(x+7)$, $x_1 = -7$, $x_2 = -10$.

3.
$$y = \begin{cases} 2x, & x \leq 1, \\ (x-1)^2, & 1 < x < 3, \\ 2-2x, & x \geq 3. \end{cases}$$

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1. а) $\lim_{x \rightarrow \infty} \frac{6x^2 + 3x}{2x^2 - 3x + 5};$

б) $\lim_{x \rightarrow 1} \frac{x^2 + x - 12}{3x - 9}; \quad \lim_{x \rightarrow 3} \frac{x^2 + x - 12}{3x - 9};$

в) $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x^2-5}-2}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1-\cos 7x}{3x^2}; \quad \text{д) } \lim_{x \rightarrow 0} \frac{\sin x - \operatorname{tg} x}{\sin^3 x};$

е) $\lim_{x \rightarrow \infty} x(\ln(2x-1) - \ln(2x+1)); \quad \lim_{x \rightarrow 1} x(\ln(2x-1) - \ln(2x+1)).$

2. $y = \frac{2-2x}{x^3-x^4}$, $x_1 = 1$, $x_2 = 0$.

3.
$$y = \begin{cases} -x, & x \leq -\frac{\pi}{4}, \\ \operatorname{tg} x, & -\frac{\pi}{4} < x \leq \frac{\pi}{4}, \\ x, & x > \frac{\pi}{4}. \end{cases}$$

$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{(x-3)^2 - (x+3)^2}{(x+2)^2};$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{x^3 + 8}{x^2 - 4}; \quad \lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 4};$$

$$\text{в) } \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{4x+1}-3}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\arcsin^2 3x}{\operatorname{tg}^2 8x}; \quad \text{д) } \lim_{x \rightarrow 0} \frac{\cos 4x - \cos 6x}{\sin^2 5x};$$

$$\text{е) } \lim_{x \rightarrow 1} (4-3x)^{\frac{x}{x-1}}; \quad \lim_{x \rightarrow 0} (4-3x)^{\frac{x}{x-1}}.$$

$$2. \quad y = x + \frac{x+2}{x^2-4}, \quad x_1 = -2, \quad x_2 = 1.$$

$$3. \quad y = \begin{cases} x-4, & x < -1, \\ x^2, & -1 \leq x < 1, \\ x+4, & x \geq 1. \end{cases}$$

$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{7x^2 - 3x}{2x^2 + 6x - 1};$$

$$\text{б) } \lim_{x \rightarrow 1} \frac{x^3 - 1}{3x - 3}; \quad \lim_{x \rightarrow 2} \frac{x^3 - 1}{3x - 3};$$

$$\text{в) } \lim_{x \rightarrow 6} \frac{\sqrt{x-5}-1}{36-x^2}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x}; \quad \text{д) } \lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{x \sin 2x};$$

$$\text{е) } \lim_{x \rightarrow 1} (7x-6)^{\frac{x+5}{2x-2}}; \quad \lim_{x \rightarrow 2} (7x-6)^{\frac{x+5}{2x-2}}.$$

$$2. \quad y = e^{\frac{1}{x+1}}, \quad x_1 = -1, \quad x_2 = 0.$$

$$3. \quad y = \begin{cases} \sin x, & x < -\pi, \\ \cos x, & -\pi \leq x < \pi, \\ 1, & x \geq \pi. \end{cases}$$

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$$1. \text{ а) } \lim_{x \rightarrow \infty} \left(\frac{x^4}{x^2 + 2} - x^2 \right);$$

$$б) \lim_{x \rightarrow 7} \frac{2x^2 + x - 1}{x^2 - 6x - 7}; \quad \lim_{x \rightarrow -1} \frac{2x^2 + x - 1}{x^2 - 6x - 7};$$

$$в) \lim_{x \rightarrow 7} \frac{\sqrt{x-3} - 2}{x^2 - 49}; \quad г) \lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x};$$

$$д) \lim_{x \rightarrow 0} (1 - \cos 5x) \cdot \operatorname{ctg}^2 3x;$$

$$е) \lim_{x \rightarrow 2} (3x - 5)^{\frac{2x}{x^2 - 4}}; \quad \lim_{x \rightarrow 3} (3x - 5)^{\frac{2x}{x^2 - 4}}.$$

$$2. \quad y = \frac{x^2}{x-2}, \quad x_1 = 2, \quad x_2 = 1.$$

$$3. \quad y = \begin{cases} -x-1, & x < 0, \\ (x+5)^2, & 0 \leq x < 3, \\ 1-x, & x \geq 3. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 1}{3x^2 + x + 4};$

б) $\lim_{x \rightarrow 2} \frac{x^2 + x - 12}{x^2 + 2x - 8}; \quad \lim_{x \rightarrow -4} \frac{x^2 + x - 12}{x^2 + 2x - 8};$

в) $\lim_{x \rightarrow -8} \frac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{2x \operatorname{tg} 2x}; \quad \text{д) } \lim_{x \rightarrow \pi} \frac{\cos \frac{x}{2}}{x - \pi};$

е) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x + 1}{x^2 - x - 2} \right)^x; \quad \lim_{x \rightarrow 3} \left(\frac{x^2 + 3x + 1}{x^2 - x - 2} \right)^x.$

2. $y = \frac{x+1}{x}, \quad x_1 = 2, \quad x_2 = 0.$

3. $y = \begin{cases} 4, & x < -\pi, \\ \cos x, & -\pi \leq x < 0, \\ 0, & x \geq 0. \end{cases}$

1. а) $\lim_{x \rightarrow \infty} \frac{5x^3 - 2x + 1}{2x^2 + x - 3};$

б) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2}; \quad \lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 2};$

в) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{1 - \sqrt[3]{1+x}}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\operatorname{arctg}^2 4x}{1 - \cos 3x}; \quad \text{д) } \lim_{x \rightarrow -1} \frac{x^3 + 1}{\sin(x+1)};$

е) $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{6x+2}{3x-6}}; \quad \lim_{x \rightarrow 0} (5 - 2x)^{\frac{6x+2}{3x-6}}.$

$$2. \quad y = \frac{x-3}{x^2-4}, \quad x_1 = 2, \quad x_2 = -2.$$

$$3. \quad y = \begin{cases} -x+3, & x < -2, \\ x^2-1, & -2 \leq x < 1, \\ 2-4x, & x \geq 1. \end{cases}$$

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$$1. \quad \text{a) } \lim_{x \rightarrow \infty} \frac{3-x^2-2x}{x^2+4x+1};$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{x^2-5x+6}{x^2-x-2}; \quad \lim_{x \rightarrow 3} \frac{x^2-5x+6}{x^2-x-2};$$

$$\text{в) } \lim_{x \rightarrow -3} \frac{\sqrt{x+4}-1}{x+3}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1-\cos 8x}{1-\cos 4x}; \quad \text{д) } \lim_{x \rightarrow \pi/2} \frac{\sqrt{1-\sin x}}{2x-\pi};$$

$$\text{е) } \lim_{x \rightarrow 2} (3-x)^{\frac{4x}{x-2}}; \quad \lim_{x \rightarrow 1} (3-x)^{\frac{4x}{x-2}}.$$

$$2. \quad y = \operatorname{arctg} \frac{1}{x-2}, \quad x_1 = 2, \quad x_2 = 3.$$

$$3. \quad y = \begin{cases} -1, & x \leq -\frac{\pi}{4}, \\ \operatorname{tg} x, & -\frac{\pi}{4} < x \leq \frac{\pi}{4}, \\ x^2, & x > \frac{\pi}{4}. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 1}{2x^4 + x + 5}$;
- б) $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x^2 - 25}$; $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x^2 - 25}$;
- в) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 1} - \sqrt{x + x^2 - 3}}{x^2 - 4}$; г) $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sqrt{x + 2} - \sqrt{2}}$;
- д) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x^2}$; е) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x - 1}{x^2 - 3x - 2} \right)^x$; $\lim_{x \rightarrow 3} \left(\frac{x^2 + 2x - 1}{x^2 - 3x - 2} \right)^x$.

2. $y = \frac{4x}{x+3}$, $x_1 = 3$, $x_2 = 0$.

3. $y = \begin{cases} -4x, & x < -1, \\ -(x-1)^2, & -1 \leq x < 1, \\ 4x, & x \geq 1. \end{cases}$

1. а) $\lim_{x \rightarrow \infty} \frac{(5x-4)(2x+2)}{5x^3 + 2x^2 + 2}$;
- б) $\lim_{x \rightarrow -1} \frac{2x^2 - 3x + 1}{x^2 - 1}$; $\lim_{x \rightarrow 1} \frac{2x^2 - 3x + 1}{x^2 - 1}$;
- в) $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{1 - \sqrt{2-x}}$; г) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{\sin 5x}$; д) $\lim_{x \rightarrow 1} (1-x) \operatorname{tg} \frac{\pi x}{2}$;
- е) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x - 1}{x^2 - 2x + 2} \right)^x$; $\lim_{x \rightarrow 1} \left(\frac{x^2 + 2x - 1}{x^2 - 2x + 2} \right)^x$.

$$2. \quad y = \frac{x+1}{x^2+x^3}, \quad x_1 = -1, \quad x_2 = 0.$$

$$3. \quad y = \begin{cases} 1+x, & x < -1, \\ x^2, & -1 \leq x < 0, \\ \sin x, & x \geq 0. \end{cases}$$

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$$1. \quad \text{a) } \lim_{x \rightarrow \infty} \frac{x^2 - 3x + 4}{2x^3 + 5x - 1};$$

$$\text{б) } \lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}; \quad \lim_{x \rightarrow -3} \frac{x^3 - 27}{x^2 - 9};$$

$$\text{в) } \lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{\sqrt{1-x} - 1}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2}; \quad \text{д) } \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{\sin(\pi(x+2))};$$

$$\text{е) } \lim_{x \rightarrow 2} (3x-5)^{\frac{2x}{x^2-4}}; \quad \lim_{x \rightarrow 3} (3x-5)^{\frac{2x}{x^2-4}}.$$

$$2. \quad y = \frac{x}{x-x^3}, \quad x_1 = 1, \quad x_2 = 0.$$

$$3. \quad y = \begin{cases} 2x+4, & x < 1, \\ 3x^2, & 1 \leq x < 3, \\ x-2, & x \geq 3. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{4x^4 - 4x^3 - 2}{5x^3 + 3x^2 - 1}$;
- б) $\lim_{x \rightarrow 0,5} \frac{2x^2 - 7x + 3}{x - 3}$; $\lim_{x \rightarrow 3} \frac{2x^2 - 7x + 3}{x - 3}$;
- в) $\lim_{x \rightarrow -1} \frac{x+1}{\sqrt{x+5}-2}$; г) $\lim_{x \rightarrow 0} x \operatorname{ctg} \frac{5x}{2}$; д) $\lim_{x \rightarrow -2} \frac{\operatorname{arctg}(x+2)}{x^2 - 4}$;
- е) $\lim_{x \rightarrow 3} (3x-8)^{\frac{8}{x-3}}$; $\lim_{x \rightarrow 4} (3x-8)^{\frac{8}{x-3}}$.

2. $y = \frac{x-1}{2x^2 - x - 1}$, $x_1 = 1$, $x_2 = \frac{1}{2}$.

3. $y = \begin{cases} x+3, & x < -\pi, \\ \sin x, & -\pi \leq x < 0, \\ 0, & x \geq 0. \end{cases}$

1. а) $\lim_{x \rightarrow \infty} \frac{x^5 - 4x^2 + 1}{3x^5 - 9x + 3}$;
- б) $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{2x^2 + 10x + 12}$; $\lim_{x \rightarrow -2} \frac{x^2 + 3x + 2}{2x^2 + 10x + 12}$;
- в) $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{\sqrt{x-1}-1}$; г) $\lim_{x \rightarrow 0} \frac{\arcsin 2x}{x}$; д) $\lim_{x \rightarrow 0} \frac{2x \sin x}{\sec x - 1}$;
- е) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x^2 + x - 1} \right)^{-x^2}$; $\lim_{x \rightarrow 2} \left(\frac{x^2 + x + 1}{x^2 + x - 1} \right)^{-x^2}$.

$$2. \quad y = \operatorname{arctg} \frac{1}{x}, \quad x_1 = 1, \quad x_2 = 0.$$

$$3. \quad y = \begin{cases} x-3, & x < 0, \\ x^2-1, & 0 \leq x < 1, \\ x+6, & x \geq 1. \end{cases}$$

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$$1. \text{ a) } \lim_{x \rightarrow \infty} \frac{6x^5 - 4x^2 + 1}{3x^5 - x + 3};$$

$$6) \lim_{x \rightarrow 7} \frac{x^2 + x - 56}{x^2 - 49}; \quad \lim_{x \rightarrow -7} \frac{x^2 + x - 56}{x^2 - 49};$$

$$b) \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{2 - \sqrt[3]{8-x}}; \quad r) \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - \cos \sqrt{x}}; \quad d) \lim_{x \rightarrow 0,5} \frac{\arcsin(1-2x)}{4x^2 - 1};$$

$$e) \lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 4}{x^2 - 3x + 7} \right)^x; \quad \lim_{x \rightarrow 0} \left(\frac{x^2 + 5x + 4}{x^2 - 3x + 7} \right)^x.$$

$$2. \quad y = \frac{1}{1 + e^{\frac{1}{x-1}}}, \quad x_1 = 1, \quad x_2 = 2.$$

$$3. \quad y = \begin{cases} \cos x, & x < 0, \\ x-1, & 0 \leq x < 2, \\ 2, & x \geq 2. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{6x^4 - 3x^5 + 1}{4x^5 - 2x + 3}$;
- б) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 2x + 1}$; $\lim_{x \rightarrow 0} \frac{x^3 - 1}{x^2 - 2x + 1}$;
- в) $\lim_{x \rightarrow 2} \frac{3 - \sqrt{7+x}}{x-2}$; г) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\operatorname{tg} x^2}$; д) $\lim_{x \rightarrow 0} \operatorname{ctg} x^2 (1 - \cos x)$;
- е) $\lim_{x \rightarrow 2} (2x-3)^{\frac{x^2}{x-2}}$; $\lim_{x \rightarrow 3} (2x-3)^{\frac{x^2}{x-2}}$.

2. $y = \ln(1+2x)$, $x_1 = -\frac{1}{2}$, $x_2 = -2$.

3. $y = \begin{cases} 4-x, & x \leq 0, \\ (x+3)^2, & 0 < x < 2, \\ 2x, & x \geq 2. \end{cases}$

1. а) $\lim_{x \rightarrow \infty} \frac{3x^4 - 2x + 1}{2x^4 + x^3 - 3}$;
- б) $\lim_{x \rightarrow -1} \frac{x^4 - 4x^2 + 3}{x-1}$; $\lim_{x \rightarrow 1} \frac{x^4 - 4x^2 + 3}{x-1}$;
- в) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{\sqrt{2x-2} - 2}$; г) $\lim_{x \rightarrow 0} \frac{\cos 7x - 1}{\sin^2 3x}$; д) $\lim_{x \rightarrow 0} \frac{\sin^2 x - \operatorname{tg}^2 x}{x^4}$;

$$\text{e) } \lim_{x \rightarrow \infty} \left(\frac{2x^2 - x + 1}{2x^2 + x + 1} \right)^{\frac{x^2}{1-x}} ; \quad \lim_{x \rightarrow 0} \left(\frac{2x^2 - x + 1}{2x^2 + x + 1} \right)^{\frac{x^2}{1-x}} .$$

$$2. \quad y = x + \frac{x-5}{x^2-25}, \quad x_1 = 5, x_2 = 0.$$

$$3. \quad y = \begin{cases} 2-x, & x \leq -\frac{\pi}{4}, \\ \operatorname{tg} x, & -\frac{\pi}{4} < x \leq 0, \\ 0, & x > 0. \end{cases}$$

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$$1. \quad \text{a) } \lim_{x \rightarrow \infty} \frac{(x+1)^3 - (x-1)^3}{x^3 + 1};$$

$$\text{б) } \lim_{x \rightarrow -1} \frac{x^2 - 5x + 6}{x^2 - 2x - 3}; \quad \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 2x - 3};$$

$$\text{в) } \lim_{x \rightarrow -2} \frac{\sqrt{x+11}-3}{x^2+2x}; \quad \text{г) } \lim_{x \rightarrow 0} \frac{1-\cos x}{x \sin x}; \quad \text{д) } \lim_{x \rightarrow 0} \frac{\sin 4x}{\sqrt{x+1}-1};$$

$$\text{e) } \lim_{x \rightarrow -2} (4x+9)^{\frac{5-x}{2+x}}; \quad \lim_{x \rightarrow 0} (4x+9)^{\frac{5-x}{2+x}}.$$

$$2. \quad y = \operatorname{arctg} \frac{1}{x-6}, \quad x_1 = 7, x_2 = 6.$$

$$3. \quad y = \begin{cases} x-1, & x < -1, \\ x^2+3, & -1 \leq x < 1, \\ -2x, & x \geq 1. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{4x^3 - 4x^2 - 2}{3x^3 + x^2 + 4x}$;
- б) $\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{3x^2 + x - 4}$; $\lim_{x \rightarrow -0,5} \frac{2x^2 - x - 1}{3x^2 + x - 4}$;
- в) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{3x}$; г) $\lim_{x \rightarrow 0} \frac{\sin 7x}{x^2 + \pi x}$; д) $\lim_{x \rightarrow 0} \frac{2x}{\operatorname{tg}(2\pi(x + 1/2))}$;
- е) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 1}{x^2 + 1} \right)^{\frac{x-1}{x+1}}$; $\lim_{x \rightarrow 2} \left(\frac{x^2 - 1}{x^2 + 1} \right)^{\frac{x-1}{x+1}}$.

2. $y = 3^{\frac{1}{1-x}}$, $x_1 = 1$, $x_2 = 0$.

3.
$$y = \begin{cases} 0, & x < -\frac{\pi}{2}, \\ \cos x, & -\frac{\pi}{2} \leq x < \pi, \\ 2, & x \geq \pi. \end{cases}$$

1. а) $\lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 + 6}{x^3 - 4x^4 + 4x}$;
- б) $\lim_{x \rightarrow -7} \frac{x^2 + 4x - 21}{x + 7}$; $\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{x + 7}$;

$$\text{в)} \lim_{x \rightarrow 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}; \quad \text{г)}; \lim_{x \rightarrow 0} \frac{2x \cdot \sin x}{1 - \cos x} \quad \text{д)} \lim_{x \rightarrow 0} \frac{x \sin 2x}{\operatorname{tg}^2 3x};$$

$$\text{е)} \lim_{x \rightarrow \infty} \left(\frac{3x^3+1}{3x^3+x^2-3} \right)^{2x+1}; \quad \lim_{x \rightarrow 1} \left(\frac{3x^3+1}{3x^3+x^2-3} \right)^{2x+1}.$$

$$2. \quad y = \frac{3}{x^2 - 2x}, \quad x_1 = 2, \quad x_2 = 0.$$

$$3. \quad y = \begin{cases} x+2, & x < 0, \\ -x^2+1, & 0 \leq x \leq 1, \\ x+5, & x > 1. \end{cases}$$