БІЛІМ АЛУШЫНЫҢ ӨЗДІК ЖҰМЫСЫНЫҢ (БӨЖ) ТАПСЫРМАЛАРЫ №7

7.1 «Анықталған интеграл және қолданбалары» тақырыбына арналған типтік есептер

1. Анықталған интегралды есептеңіз:

1	а) $\int_{-1}^{2}(x^4-3x^2)dx$	ә) $\int_{0}^{\pi} x\sin x dx$	б) $\int_{2}^{3}\frac{x}{\sqrt{x^2+1}}dx$
2	а) $\int_{0}^{1}\sqrt{5x+4}dx$	ә) $\int_{0}^{\pi} x\cos x dx$	б) $\int_{2}^{4}\frac{x}{\sqrt{x^2-2}}dx$
3	а) $\int_{3}^{10}\sqrt[3]{x-2}dx$	ә) $\int_{0}^{1} x\cdot e^{2x}dx$	б) $\int_{1}^{3}\frac{x-1}{\sqrt{x^2+3}}dx$
4	а) $\int_{0}^{\pi}\sin^2\frac{x}{4}dx$	ә) $\int_{0}^{1} x\cdot\cos\frac{\pi x}{4}dx$	б) $\int_{1}^{2}\frac{2+3x}{\sqrt{x^2+10}}dx$
5	а) $\int_{-1}^{-0,5}\frac{x^2-1}{x^4}dx$	ә) $\int_{0}^{\frac{1}{2}} x\cdot\sin\frac{\pi x}{3}dx$	б) $\int_{1}^{3}\frac{8x+7}{\sqrt{9+x^2}}dx$
6	а) $\int_{-1}^{0}\frac{dx}{\sqrt{x+5}}$	ә) $\int_{0}^{\frac{1}{3}} x\cdot\cos\pi x dx$	б) $\int_{1}^{2}\frac{4x-7}{\sqrt{x^2+24}}dx$
7	а) $\int_{-\frac{\pi}{4}}^{0}\sin^2 2tdt$	ә) $\int_{1}^{e}\frac{\ln x}{x^3}dx$	б) $\int_{0}^{3}\frac{x+4}{\sqrt{x^2+16}}dx$
8	а) $\int_{3.5}^{14}\sqrt[3]{2x+1}dx$	ә) $\int_{0}^{1} arctgx dx$	б) $\int_{1}^{3}\frac{2x+8}{\sqrt{10-x^2}}dx$
9	а) $\int_{1}^{8}\frac{dx}{\sqrt[3]{x^2}}.$	ә) $\int_{e}^{e^2}\sqrt{x}\cdot\ln x dx$	б) $\int_{3}^{7}\frac{x+1}{\sqrt{x^2-4}}dx$
10	а) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin 2x dx$	ә) $\int_{0}^{\frac{\pi}{4}}\frac{xdx}{\cos^2 x}$	б) $\int_{3}^{7}\frac{x-2}{\sqrt{5-x^2}}dx$
11	а) $\int_{-\frac{\pi}{8}}^{0}\frac{dx}{\cos^2 2x}$	ә) $\int_{0}^{\frac{\pi}{2}} x\cos x dx$	б) $\int_{3}^{7}\frac{2x+1}{\sqrt{x^2+6}}dx$

12	а) $\int_0^1 \frac{x+2}{x+1}dx$	ə) $\int_0^{\frac{\pi}{4}} x\cdot\sin 2xdx$	б) $\int_0^2 \frac{3x+4}{\sqrt{7-x^2}}dx$
13	а) $\int_{0,5}^1 e^{-2x+1}dx.$	ə) $\int_0^{\pi} x\cdot\cos 2xdx$	б) $\int_1^4 \frac{2x-5}{\sqrt{x^2+8}}dx$
14	а) $\int_1^e \frac{\ln x}{x}dx$	ə) $\int_0^{\pi} x\cdot\sin\frac{x}{3}dx$	б) $\int_1^2 \frac{3x+5}{\sqrt{9-x^2}}dx$
15	а) $\int_0^{\frac{\pi}{4}} \sin 5xdx$	ə) $\int_{-1}^0 x\cdot e^{-x}dx$	б) $\int_0^1 \frac{4-3x}{\sqrt{3x^2+7}}dx$
16	а) $\int_{-1}^1 \frac{dx}{x^2+2x+5}$	ə) $\int_0^{\pi} x\cos\frac{x}{2}dx$	б) $\int_{-2}^2 \frac{4x-3}{\sqrt{x^2+11}}dx$
17	а) $\int_{-2}^{-1} \frac{dx}{x^2}$	ə) $\int_{-21}^0 x^2\cdot e^{\frac{x}{2}}dx$	б) $\int_0^2 \frac{x-2}{\sqrt{x^2+12}}dx$
18	а) $\int_{0,5}^1 \frac{dx}{(x+1)^2}$	ə) $\int_{-\frac{\pi}{2}}^{-\frac{\pi}{4}} \frac{xdx}{\sin^2 x}$	б) $\int_0^4 \frac{3x-7}{\sqrt{9+x^2}}dx$
19	а) $\int_{-\frac{\pi}{4}}^0 \cos^2 tdt$	ə) $\int_0^{\pi/4} x\cos 2xdx$	б) $\int_2^3 \frac{4-x}{\sqrt{x^2+14}}dx$
20	а) $\int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2(\frac{x}{2})}$	ə) $\int_0^{\pi} x\cdot\sin\frac{x}{2}dx$	б) $\int_1^2 \frac{7x-2}{\sqrt{2x^2+3}}dx$
21	а) $\int_{-1}^{-0,5} \frac{x^2-1}{x^4}dx$	ə) $\int_0^1 x\cdot\cos(\frac{\pi}{3}x)dx$	б) $\int_1^2 \frac{2-x}{\sqrt{x^2+10}}dx$
22	а) $\int_{-1}^0 \frac{dx}{\sqrt{x+2}}$	ə) $\int_0^{\frac{1}{2}} x\cdot\sin(\frac{\pi}{2}x)dx$	б) $\int_1^3 \frac{8x-9}{\sqrt{9+x^2}}dx$
23	а) $\int_{-0.5\pi}^0 \sin^2 tdt$	ə) $\int_0^{0,5} x\cos(\pi x)dx$	б) $\int_1^2 \frac{5x+6}{\sqrt{x^2-18}}dx$
24	а) $\int_{2,5}^3 \frac{dx}{(x-2)^3}.$	ə) $\int_1^e \frac{\ln x}{x^2}dx$	б) $\int_0^3 \frac{2-x}{\sqrt{x^2+16}}dx$

25	а) $\int_{7}^{26}\sqrt[3]{x+1}dx$	ə) $\int_{-e}^{-1} x\cdot\ln x dx$	б) $\int_{0}^{4}\frac{2x-8}{\sqrt{10+x^2}}dx$
26	а) $\int_{1}^{3}\frac{dx}{x^2-2x+5}$	ə) $\int_{0}^{\pi} x\sin\frac{x}{2}dx$	б) $\int_{4}^{6}\frac{4x+3}{\sqrt{x^2-11}}dx$
27	а) $\int_{-2}^{-1}\frac{dx}{x^3}$	ə) $\int_{-3}^{0} x\cdot e^{\frac{x}{3}}dx$	б) $\int_{4}^{6}\frac{x+2}{\sqrt{x^2-12}}dx$
28	а) $\int_{-2}^{-1}\frac{dx}{(x-1)^2}$	ə) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{xdx}{\cos^2 x}$	б) $\int_{0}^{1}\frac{3x+7}{\sqrt{9-x^2}}dx$
29	а) $\int_{-\frac{\pi}{4}}^{0}\cos^2 2tdt$	ə) $\int_{0}^{\frac{\pi}{8}} x\cdot\cos 4xdx$	б) $\int_{4}^{8}\frac{4+x}{\sqrt{x^2-14}}dx$
30	а) $\int_{0}^{\frac{\pi}{2}}\frac{dx}{\cos^2\frac{x}{3}}$	ə) $\int_{0}^{\pi} x\cdot\sin\frac{x}{4}dx$	б) $\int_{2}^{3}\frac{7x+2}{\sqrt{2x^2-3}}dx$
31	а) $\int_{-1}^{-0,5}\frac{x^2-1}{x^4}dx$	ə) $\int_{0}^{1} x\cdot\cos(\frac{\pi}{3}x)dx$	б) $\int_{1}^{2}\frac{2-x}{\sqrt{x^2+9}}dx$
32	а) $\int_{-1}^{0}\frac{dx}{\sqrt{x+2}}$	ə) $\int_{0}^{\frac{1}{2}} x\cdot\sin(\frac{\pi}{2}x)dx$	б) $\int_{1}^{3}\frac{8x-14}{\sqrt{9+x^2}}dx$
33	а) $\int_{-0.5\pi}^{0}\sin^2 tdt$	ə) $\int_{0}^{0,5} x\cos(\pi x)dx$	б) $\int_{1}^{2}\frac{5x+6}{\sqrt{x^2-16}}dx$
34	а) $\int_{2,5}^{3}\frac{dx}{(x-2)^3}$.	ə) $\int_{1}^{e}\frac{\ln x}{x^2}dx$	б) $\int_{0}^{3}\frac{2-x}{\sqrt{x^2+25}}dx$
35	а) $\int_{7}^{26}\sqrt[3]{x+1}dx$	ə) $\int_{-e}^{-1} x\cdot\ln x dx$	б) $\int_{0}^{4}\frac{2x-8}{\sqrt{10+x^2}}dx$
36	а) $\int_{1}^{3}\frac{dx}{x^2-2x+5}$	ə) $\int_{0}^{\pi} x\sin\frac{x}{2}dx$	б) $\int_{4}^{6}\frac{4x+3}{\sqrt{x^2-11}}dx$
37	а) $\int_{-2}^{-1}\frac{dx}{x^3}$	ə) $\int_{-3}^{0} x\cdot e^{\frac{x}{3}}dx$	б) $\int_{4}^{6}\frac{x+2}{\sqrt{x^2-12}}dx$

38	а) $\int_{-2}^{-1} \frac{dx}{(x-1)^2}$	ә) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{xdx}{\cos^2 x}$	б) $\int_0^1 \frac{3x+7}{\sqrt{9-x^2}}dx$
39	а) $\int_{-\frac{\pi}{4}}^{0} \cos^2 2tdt$	ә) $\int_0^{\frac{\pi}{8}} x \cdot \cos 4xdx$	б) $\int_4^8 \frac{3+x}{\sqrt{x^2-112}}dx$
40	а) $\int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2 \frac{x}{3}}$	ә) $\int_0^{\pi} x \cdot \sin \frac{x}{2}dx$	б) $\int_2^3 \frac{5x+2}{\sqrt{2x^2-6}}dx$

2. Көрсетілген айнымалыны ауыстыруды қолданып, интегралды есептеңіз:

1	$\int_0^2 x^3 \cdot e^{x^2}dx, \ / x^2 = t /$	2	$\int_0^{0.5} x^3 \cdot e^{-x^4}dx, \ / x^4 = t /$
3	$\int_1^{\sqrt{2}} x^3 \cdot 2^{-x^2}dx, \ / x^2 = t /$	4	$\int_0^{\frac{\pi}{2}} \frac{dx}{4-3\cos x}, \ / tg\frac{x}{2} = t /$
5	$\int_1^{\ln^2 2} e^{2\sqrt{x}}dx, \ / \sqrt{x} = t /$	6	$\int_0^1 x^3 \cdot e^{4x^2}dx, \ / x^2 = t /$
7	$\int_0^{\sqrt{2}} \frac{xdx}{x^4+2x^2+4}, \ / x^2 = t /$	8	$\int_0^1 x \cdot e^{3\sqrt{x}}dx, \ / \sqrt{x} = t /$
9	$\int_2^3 \frac{dx}{1+\sqrt{2x-1}}, \ / 2x-1 = t^2 /$	10	$\int_1^e \frac{dx}{x \cdot (\ln^2 x+1)}, \ /\ln x = t/$
11	$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\cos x}, / tg\frac{x}{2} = t /$	12	$\int_1^6 \frac{dx}{x+\sqrt{3x-2}}, \ \left\|3x-2=t^2\right\|$
13	$\int_4^9 e^{\sqrt{x}}dx, \quad / x = t^2 /$	14	$\int_0^2 \sqrt{4-x^2}dx, \quad / x = 2\sin t /$
15	$\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x}dx, / x = \frac{1}{\cos t} /$	16	$\int_{\ln 1}^{\ln 3} \frac{dx}{e^x+e^{-x}} \ / e^x = t /$
17	$\int_{-1}^1 \frac{xdx}{\sqrt{5+4x^2}}, \ / 5+4x^2 = t^2 /$	18	$\int_e^{e^2} \frac{dx}{x \cdot \ln^3 x}, \ / \ln x = t /$

19	$\int_0^4 \sqrt{16-x^2}\,dx, / x=4\sin t /$	20	$\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx, \ / \sqrt{x}=t /$
21	$\int_1^{\ln^2 2} e^{-\sqrt{x}}dx, \ / \sqrt{x}=t /$	22	$\int_0^{\frac{\pi}{2}} \frac{dx}{4+3\cos x}, \ / tg\frac{x}{2}=t /$
23	$\int_0^4 x\cdot e^{-\sqrt{x}}dx, \ / \sqrt{x}=t /$	24	$\int_0^1 x^3\cdot e^{x^2}dx, \ / x^2=t /$
25	$\int_0^{\sqrt{2}} \frac{xdx}{x^4+2x^2+25}, \ / x^2=t /$	26	$\int_0^{\frac{\pi}{2}} \frac{dx}{3-2\cos x}, / tg\frac{x}{2}=t /$
27	$\int_0^1 \frac{xdx}{\sqrt{5-4x^2}}, /5-4x^2=t^2 /$	28	$\int_e^{e^2} \frac{dx}{x\cdot \ln^5 x}, \ / \ln x=t /$
29	$\int_0^3 \sqrt{9-x^2}\,dx, / x=3\sin t /$	30	$\int_0^1 \frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx, \ / \sqrt{x}=t /$
31	$\int_1^{\ln^2 2} e^{-\sqrt{x}}dx, \ / \sqrt{x}=t /$	32	$\int_0^{\frac{\pi}{2}} \frac{dx}{4+3\cos x}, \ / tg\frac{x}{2}=t /$
33	$\int_0^4 x\cdot e^{-\sqrt{x}}dx, \ / \sqrt{x}=t /$	34	$\int_0^1 x^3\cdot e^{x^2}dx, \ / x^2=t /$
35	$\int_0^{\sqrt{2}} \frac{xdx}{x^4+2x^2+25}, \ / x^2=t /$	36	$\int_0^{\frac{\pi}{2}} \frac{dx}{3-2\cos x}, / tg\frac{x}{2}=t /$
37	$\int_0^1 \frac{xdx}{\sqrt{5-4x^2}}, /5-4x^2=t^2 /$	38	$\int_e^{e^2} \frac{dx}{x\cdot \ln^5 x}, \ / \ln x=t /$
39	$\int_0^3 \sqrt{9-x^2}\,dx, / x=3\sin t /$	40	$\int_0^1 \frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx, \ / \sqrt{x}=t /$

3. Төмендегі сызықтармен шектелген фигураның ауданын есептеңіз.

1	$y=x^2, \ y=4$	2	$y=\sqrt{x}, \ y=1, y=0, \ x=2$
3	$y=tgx, \quad y=1/\sqrt{3}, \ x=0$	4	$y=\sin^2 x, \quad y=\sin x, \ 0\le x\le \pi$
5	$y=\frac{11}{2+x^2}, \quad y=1$	6	$y=\frac{1}{x}, \quad y=1, \ y=0, \ x=2$
7	$y=x^2, \quad y=x$	8	$y=arctgx \quad y=\pi/3 \quad x=0$

9	$y=\frac{3}{x},\quad x+y=4$	10	$y=e^{-x},\quad y=1,\quad x=\ln 3$
11	$y=tg2x,\ y=-1,\quad x=0$	12	$y=\cos x,\quad y=0,\quad -\pi/2\le x\le \pi/2$
13	$y=\frac{5}{1+x^2},\quad y=1$	14	$y=\sin^2 x,\quad y=0,\quad 0\le x\le \pi/2$
15	$y=x^2,\quad y=-x^2,\quad x=1$	16	$y=tgx,\quad y=1,\quad x=0$
17	$y=tgx,\quad y=-\sqrt{3},\quad x=0$	18	$y=\sin^2 x,\quad y=0,\quad 0\le x\le \pi$
19	$y=x^2,\quad y=x^4$	20	$y=\cos^2 x,\ y=\cos x,\ \lvert x\rvert\le \pi/2$
21	$y=tg2x,\quad y=1,\quad x=0$	22	$y=arctgx,\quad y=\pi/4,\quad x=0$
23	$y=x^2-4,\quad y=x+2$	24	$y=tgx,\quad y=\sqrt{3},\quad x=0$
25	$y=ctg3x,\quad y=1,\ x=\frac{\pi}{6}$	26	$y=tg2x,\quad y=\sqrt{3},\quad x=0$
27	$y=x^2,\quad y=\sqrt{x}$	28	$y=tgx,\quad y=\sqrt{3},\quad x=0$
29	$y=ctg2x,\quad y=1,\ x=\frac{\pi}{4}$	30	$y=\cos^2 x,\ y=0,\quad 0\le x\le \pi/2$
31	$y=tg2x,\quad y=1,\quad x=0$	32	$y=arctgx,\quad y=\pi/4,\quad x=0$
33	$y=x^2-4,\quad y=x+2$	34	$y=tgx,\quad y=\sqrt{3},\quad x=0$
35	$y=ctg3x,\quad y=1,\ x=\frac{\pi}{6}$	36	$y=tg2x,\quad y=\sqrt{3},\quad x=0$
37	$y=x^2,\quad y=\sqrt{x}$	38	$y=tgx,\quad y=\sqrt{3},\quad x=0$
39	$y=ctg2x,\quad y=1,\ x=\frac{\pi}{4}$	40	$y=tgx,\quad y=-\sqrt{3},\quad x=0$

4. Меншіксіз интегралды есептеңіз:

1	$\int_0^{+\infty}\frac{dx}{3x^2+1}$	2	$\int_0^{+\infty}\frac{dx}{x^2-4x+13}$	3	$\int_3^{+\infty}\frac{dx}{x+1}$
4	$\int_{-\infty}^{+\infty}\frac{dx}{x^2-6x+13}.$	5	$\int_1^{\infty}\frac{1}{x^2+8x+12}dx.$	6	$\int_0^{+\infty}\frac{x}{3x^2+1}dx$
7	$\int_0^{+\infty}\frac{x}{x^2+2x+2}dx$	8	$\int_1^{+\infty}\frac{dx}{x^2+6x+8}$	9	$\int_1^{+\infty}\frac{dx}{x(x+5)}$

10	$\int_{-\infty}^{+\infty} \frac{dx}{x^2+4x+13}$	11	$\int_{1}^{+\infty} \frac{dx}{x\cdot(x+1)}$	12	$\int_{-1}^{+\infty} \frac{xdx}{4x^2+25}$
13	$\int_{6}^{+\infty} \frac{x}{x^2+2x+50}dx$	14	$\int_{0}^{+\infty} \frac{x}{3x^2+5}dx$	15	$\int_{1}^{+\infty} \frac{dx}{2x+5}$
16	$\int_{-1}^{+\infty} \frac{dx}{x^2-4x+9}$	17	$\int_{1}^{\infty} \frac{dx}{x^2-8x+25}$	18	$\int_{1}^{+\infty} \frac{dx}{x+1}dx$
19	$\int_{1}^{+\infty} \frac{dx}{(x+1)(x+2)}$	20	$\int_{-1}^{+\infty} \frac{xdx}{x^2+9}$	21	$\int_{1}^{\infty} \frac{1}{x^2-8x+15}dx$
22	$\int_{0}^{+\infty} \frac{xdx}{x^2+4x+20}$	23	$\int_{1}^{\infty} \frac{dx}{x^2-8x+25}.$	24	$\int_{1}^{+\infty} \frac{dx}{x+1}dx$
25	$\int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+5}$	26	$\int_{1}^{+\infty} \frac{dx}{x^2+2x+50}$	27	$\int_{0}^{+\infty} \frac{x}{3x^2+1}dx$
28	$\int_{-1}^{+\infty} \frac{dx}{x^2+8x+25}$	29	$\int_{1}^{+\infty} \frac{dx}{4x+7}$	30	$\int_{0}^{+\infty} \frac{dx}{x^2-4x+13}$
31	$\int_{0}^{+\infty} \frac{x}{4x^2+3}dx$	32	$\int_{1}^{+\infty} \frac{dx}{x^2+6x+8}$	33	$\int_{0}^{+\infty} \frac{x}{x^2+2x+2}dx$
34	$\int_{1}^{+\infty} \frac{dx}{x\cdot(x+1)}$	35	$\int_{-1}^{+\infty} \frac{xdx}{4x^2+25}$	36	$\int_{1}^{\infty} \frac{1}{x^2-8x+15}dx$
37	$\int_{0}^{+\infty} \frac{xdx}{x^2+4x+20}$	38	$\int_{1}^{+\infty} \frac{dx}{x(x+8)}$	39	$\int_{1}^{+\infty} \frac{dx}{x^2+6x+9}$
40	$\int_{-1}^{+\infty} \frac{xdx}{x^2+16}$				

7.2 БӨЖ орындау үлгісі

1. Анықталған интегралдарды есептеңіз.

Шешімі:

а) $\int_{\pi/6}^{\pi/4} \frac{dx}{\cos^2 x} = tgx\Big|_{\pi/6}^{\pi/4} = tg\frac{\pi}{4} - tg\frac{\pi}{6} = 1 - \frac{\sqrt{3}}{3}.$

ә) $$\int_0^{\pi} x\sin x\,dx = \begin{vmatrix} x=u;\ dx=du \\ \sin x dx = dv; \\ -\cos x = v \end{vmatrix} = -x\cos x\Big|_0^{\pi} + \int_0^{\pi}\cos x dx = -\pi\cos\pi + \sin x\Big|_0^{\pi} = \pi.$$

б) $$\int_6^8 \frac{x+7}{\sqrt{100-x^2}}dx = \int_6^8 \frac{xdx}{\sqrt{100-x^2}} + 7\int_6^8 \frac{dx}{\sqrt{100-x^2}} = -\sqrt{100-x^2}\Big|_6^8 + \frac{7}{10}\arcsin\frac{x}{10}\Big|_6^8 =$$

$$= -\sqrt{100-64} + \sqrt{100-36} + \frac{7}{10}\left(\arcsin\frac{8}{10} - \arcsin\frac{6}{10}\right) = 2 + \frac{7}{10}(\arcsin 0.8 - \arcsin 0.6).$$

2. Көрсетілген айнымалыны ауыстыруды қолданып, интегралды есептеңіз.

Шешімі:

$$\int_0^1 \sqrt{1-x^2}\,dx = \begin{vmatrix} x=\sin x & dx=\cos x dx \\ x=0 & t=0 \\ x=1 & t=\frac{\pi}{2} \end{vmatrix} = \int_0^{\pi/2}\sqrt{1-\sin^2 t}\cos t dt = \int_0^{\pi/2}\cos^2 t\,dt =$$

$$= \frac{1}{2}\int_0^{\pi/2}(1+\cos 2t)dt = \frac{1}{2}\left(t + \frac{1}{2}\sin 2x\right)\Big|_0^{\pi/2} = \frac{\pi}{4} + \frac{1}{4}\sin\pi = \frac{\pi}{4}.$$

3. $y = x^5;\ y = 32$ сызықтарымен және ординат өсімен шектелген фигура ауданын табыңыз.

Шешімі: Берілген сызықтардың қиылысу нүктесін табамыз.

$$y = x^5;\ y = 32 \Rightarrow x = 2.$$

Берілген фигура сол жағынан ордината өсімен шектелгендіктен, төменгі шек $x = 0$ болады. Сонымен формула бойынша

$$S = \int_a^b (f_2(x) - f_1(x))dx = \int_0^2 (32 - x^5)dx = (32x - \frac{x^6}{6})\Big|_0^2 = 64 - \frac{32}{3} = \frac{64}{3}.$$

4. Меншіксіз интегралды есептеңіз:

$$\int_0^{\infty}\frac{dx}{x^2+2x+2} = \lim_{b\to\infty}\int_0^b \frac{dx}{(x+1)^2+1} = \lim_{b\to\infty} arctg(x+1)\Big|_0^b = \lim_{b\to\infty}\left(arctg(b+1) - arctg1\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

Интеграл жинақты.