# БІЛІМ АЛУШЫНЫҢ ӨЗДІК ЖҰМЫСЫНЫҢ (БӨЖ) ТАПСЫРМАЛАРЫ №6

## 6.1 «Анықталмаған интеграл» тақырыбына арналған типтік есептер

**1.** Анықталмаған интегралдарды табыңыз:

| | | |
|---|---|---|
| 1 | а) $\int\left(\frac{1-5x^3}{x^4}+\frac{1}{x-5}\right)dx$ | ә) $\int\left(\frac{1}{x^2-4}+\frac{3}{\sqrt{9-x^2}}\right)dx$ |
| 2 | а) $\int\frac{\sqrt{x}-3x\cdot\sqrt[3]{x}+x^5}{x^2}dx$ | ә) $\int\left(\frac{4}{5+x^2}+\frac{7}{\sqrt{x^2+9}}\right)dx$ |
| 3 | а) $\int x\cdot\sqrt[3]{a\cdot x}dx$ | ә) $\int\left(\frac{6}{5-x^2}+\frac{9}{\sqrt{10-x^2}}\right)dx$ |
| 4 | а) $\int\frac{(\sqrt{x}-2\sqrt[3]{x})^2}{x}dx$ | ә) $\int\left(\frac{4}{1-7x^2}+\frac{6}{\sqrt{10+x^2}}\right)dx$ |
| 5 | а) $\int\left(\frac{3}{\sqrt[3]{x^2}}-\frac{x+2}{\sqrt[4]{x}}\right)dx$ | ә) $\int\left(\frac{1}{3+7x^2}+\frac{5}{\sqrt{x+9}}\right)dx$ |
| 6 | а) $\int(a^{\frac{1}{3}}-x^{\frac{1}{3}})^3dx$ | ә) $\int\left(\frac{1}{3-2x^2}+\frac{2}{\sqrt{5-2x^2}}\right)dx$ |
| 7 | а) $\int(5x^3-\frac{\sin 5}{\sqrt[3]{x^5}})dx$ | ә) $\int\left(\frac{1}{\sqrt{4x+7}}+\frac{2}{\sqrt{5+4x^2}}\right)dx$ |
| 8 | а) $\int\left(\frac{x^2}{\sqrt{x}}+\frac{a}{x^2}-x\right)dx$ | ә) $\int\left(\frac{2}{\sqrt{1-4x}}+\frac{6}{\sqrt{5-2x^2}}\right)dx$ |
| 9 | а) $\int\frac{(\sqrt{x^3}-2x^2+3x^4)}{x}dx$ | ә) $\int\left(\frac{8}{5-3x^2}-\frac{2}{\sqrt[3]{(x+2)^4}}\right)dx$ |
| 10 | а) $\int\left(\sqrt{x}-\frac{1}{\sqrt[3]{x}}\right)^2dx$ | ә) $\int\left(\frac{7}{5x^2-3}-\frac{2}{\sqrt{3x^2+5}}\right)dx$ |
| 11 | а) $\int(\sqrt{x}+\frac{1}{\sqrt[3]{x}})^3dx$ | ә) $\int\left(\frac{5}{2+3x^2}-\frac{3}{\sqrt{x^2-4}}\right)dx$ |
| 12 | а) $\int(x\sin a-\frac{4\sqrt{x}}{x^2})dx$ | ә) $\int\left(\frac{7}{\sqrt[4]{(x-2)^5}}+\frac{6}{\sqrt{16-x^2}}\right)dx$ |
| 13 | а) $\int\frac{(\sqrt{x^5}+2x^3-x^5)}{x}dx$ | ә) $\int\left(\frac{3}{x^2-5}-\frac{1}{\sqrt{x^2-3}}\right)dx$ |

| | | |
|---|---|---|
| 14 | a) $\int \frac{x^3 - 3\sqrt{x^5} + \sqrt[6]{x}}{\sqrt{x}} dx$ | ə) $\int \left( \frac{8}{5+4x^2} + \frac{5}{\sqrt{3x^2-4}} \right) dx$ |
| 15 | a) $\int x^2 \cdot \sqrt[4]{b \cdot x} dx$ | ə) $\int \left( \frac{4}{4x^2+1} - \frac{3}{\sqrt{x^2-4}} \right) dx$ |
| 16 | a) $\int \frac{x^4 - 2x - \sqrt{x}}{x^2} dx$ | ə) $\int \left( \frac{3}{4-x^2} - \frac{4}{16+x^2} \right) dx$ |
| 17 | a) $\int \frac{\left( \sqrt{b \cdot x} - \frac{x}{b} \right)^2}{x} dx$ | ə) $\int \left( \frac{7}{9-x^2} + \frac{1}{\sqrt{2x^2+8}} \right) dx$ |
| 18 | a) $\int \frac{(x-1)^3}{x \cdot \sqrt{x}} dx$ | ə) $\int \left( \frac{1}{25+4x^2} - \frac{1}{\sqrt{4-x^2}} \right) dx$ |
| 19 | a) $\int \frac{(2x+a)^3}{x^2} dx$ | ə) $\int \left( \frac{8}{16-27x^2} - \frac{5}{\sqrt{1+5x^2}} \right) dx$ |
| 20 | a) $\int \frac{(x-a)^2}{\sqrt{x}} dx$ | ə) $\int \left( \frac{5}{63+4x^2} - \frac{6}{\sqrt{1+2x^2}} \right) dx$ |
| 21 | a) $\int (\sqrt{x} - \frac{1}{\sqrt[3]{x}})^3 dx$ | ə) $\int \left( \frac{6}{2-3x^2} + \frac{4}{\sqrt{x^2+4}} \right) dx$ |
| 22 | a) $\int (x^2 + \frac{4\sqrt{x}}{x^2}) dx$ | ə) $\int \left( \frac{7}{\sqrt[4]{(x+2)^5}} + \frac{6}{\sqrt{16+x^2}} \right) dx$ |
| 23 | a) $\int \frac{(\sqrt{x^3} - 3x^3 + x^5)}{x^2} dx$ | ə) $\int \left( \frac{3}{x^2+5} + \frac{2}{x^2-3} \right) dx$ |
| 24 | a) $\int \frac{x^3 - 3\sqrt[3]{x^5} + \sqrt[6]{x}}{\sqrt{x}} dx$ | ə) $\int \left( \frac{8}{5-4x^2} + \frac{6}{\sqrt{3x^2+4}} \right) dx$ |
| 25 | a) $\int x^3 \cdot \sqrt[5]{b \cdot x} dx$ | ə) $\int \left( \frac{4}{4x^2-1} + \frac{5}{\sqrt{x^2+4}} \right) dx$ |
| 26 | a) $\int \frac{x^4 + 2x + \sqrt{x}}{x^3} dx$ | ə) $\int \left( \frac{3}{4+x^2} + \frac{1}{\sqrt{x^2-4}} \right) dx$ |
| 27 | a) $\int \frac{(\sqrt{x^3} + x)^2}{x} dx$ | ə) $\int \left( \frac{7}{9+x^2} + \frac{4}{25-x^2} \right) dx$ |
| 28 | a) $\int \frac{(x+1)^3}{x^2 \sqrt{x}} dx$ | ə) $\int \left( \frac{3}{25-4x^2} + \frac{1}{\sqrt{4+x^2}} \right) dx$ |
| 29 | a) $\int \frac{(3x-a)^3}{x^2} dx$ | ə) $\int \left( \frac{4}{16+27x^2} - \frac{4}{\sqrt{1-5x^2}} \right) dx$ |

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|---|---|---|
| 30 | а) $\int(\sqrt{x}+\frac{1}{\sqrt[3]{x}})^2 dx$ | ә) $\int\left(\frac{5}{63-4x^2}-\frac{4}{\sqrt[5]{(x+4)^8}}\right)dx$ |
| 31 | а) $\int(\sqrt{x}-\frac{1}{\sqrt[3]{x}})^3 dx$ | ә) $\int\left(\frac{6}{2-5x^2}+\frac{5}{\sqrt{x^2+9}}\right)dx$ |
| 32 | а) $\int(x^2+\frac{3\sqrt{x}}{x^3})dx$ | ә) $\int\left(\frac{9}{\sqrt[4]{(x+2)^5}}+\frac{8}{\sqrt{25+x^2}}\right)dx$ |
| 33 | а) $\int\frac{(\sqrt{x^3}-8x^3+x^4)}{x^2}dx$ | ә) $\int\left(\frac{12}{x^2+6}+\frac{2}{x^2-9}\right)dx$ |
| 34 | а) $\int\frac{x^2-3\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{x}}dx$ | ә) $\int\left(\frac{3}{5-3x^2}+\frac{5}{\sqrt{3x^2+6}}\right)dx$ |
| 35 | а) $\int x^5\cdot\sqrt[5]{b\cdot x}dx$ | ә) $\int\left(\frac{3}{6x^2-1}+\frac{7}{\sqrt{x^2+4}}\right)dx$ |
| 36 | а) $\int\frac{x^3+8x+\sqrt{x}}{x^3}dx$ | ә) $\int\left(\frac{2}{9+x^2}+\frac{1}{\sqrt{x^2-9}}\right)dx$ |
| 37 | а) $\int\frac{(\sqrt{x^4}+x)^2}{x}dx$ | ә) $\int\left(\frac{7}{16+x^2}+\frac{2}{16-x^2}\right)dx$ |
| 38 | а) $\int\frac{(x+1)^3}{x\sqrt{x}}dx$ | ә) $\int\left(\frac{7}{16-4x^2}+\frac{1}{\sqrt{9+x^2}}\right)dx$ |
| 39 | а) $\int\frac{(5x-a)^3}{x^3}dx$ | ә) $\int\left(\frac{2}{4+27x^2}-\frac{3}{\sqrt{1-5x^2}}\right)dx$ |
| 40 | а) $\int(2\sqrt{x}+\frac{1}{\sqrt{x}})^2 dx$ | ә) $\int\left(\frac{3}{63-4x^2}-\frac{2}{\sqrt[5]{(x+2)^3}}\right)dx$ |

**2.** Анықталмаған интегралдарды табыңыз

| | | | |
|---|---|---|---|
| 1 | $\int(5\cdot\cos x-\frac{7}{\sin^2 x})dx)$ | 2 | $\int\sin 2x\cdot\cos^3 2xdx$ |
| 3 | $\int(3\cdot\sin x+\frac{2}{\cos^2 x})dx$ | 4 | $\int\frac{2+\sin^3 x}{\sin^2 x}dx$ |
| 5 | $\int\frac{\sin^2 x-\frac{5}{\sin x}}{\sin x}dx$ | 6 | $\int(\cos x-\sin x)^2 dx$ |
| 7 | $\int\frac{5\cdot\sin 2x-5\cdot tgx}{\sin x\cdot\cos x}dx$ | 8 | $\int(6\cos 4x+4\sin 6x)dx$ |

| | | | |
|---|---|---|---|
| 9 | $\int \frac{\cos 2x - 5\cdot\cos^3 x}{\cos^2 x}dx)$ | 10 | $\int (4\sin x + \frac{5}{\cos^2 x})dx$ |
| 11 | $\int \frac{1+5\cdot\cos^3 x}{\cos 2x + \sin^2 x}dx)$ | 12 | $\int (2tg3x - \frac{4}{\cos^2 x})dx$ |
| 13 | $\int \frac{1-3\sin^3 x}{\sin^2 x}dx$ | 14 | $\int \frac{2+tg^2 3x}{\sin^2 3x}dx$ |
| 15 | $\int \frac{2+\cos^3 x}{\cos^2 x}dx$ | 16 | $\int (\frac{\sin x}{4} + \frac{5}{\cos^2 x})dx$ |
| 17 | $\int (5\cdot\sin x - \frac{4}{\cos^2 x})dx$ | 18 | $\int \frac{tg2x}{\sin x\cdot\cos x}dx$ |
| 19 | $\int (2tg3x - \frac{4}{\sin^2 2x})dx$ | 20 | $\int \frac{ctgx}{\sin 2x}dx$ |
| 21 | $\int \frac{5+\sin 2x}{\sin^2 x}dx$ | 22 | $\int \sin 4x\cdot\cos^3 4x dx$ |
| 23 | $\int \frac{\cos 2x}{\cos^2 x}dx$ | 24 | $\int \frac{7+\sin^3 x}{\sin^2 x}dx$ |
| 25 | $\int (\cos x + \sin x)^2 dx$ | 26 | $\int (\cos x - \sin x)^2 dx$ |
| 27 | $\int (3\cdot\cos x - 4\cdot\sin x)dx$ | 28 | $\int (6\cos 3x + 4\sin 9x)dx$ |
| 29 | $\int (5\cdot\sin x - \frac{4}{\cos^2 x})dx$ | 30 | $\int (\sin 2x + \frac{7}{\cos^2 x})dx$ |
| 31 | $\int \frac{4-5\cos^3 x}{\cos^2 x}dx$ | 32 | $\int (5tg2x - \frac{3}{\cos^2 x})dx$ |
| 33 | $\int \frac{\cos 2x - \cos^2 x}{\sin^4 x}dx$ | 34 | $\int \frac{2+tg^2 4x}{\sin^2 4x}dx$ |
| 35 | $\int (\frac{\sin x}{2} + \frac{7}{\cos^2 x})dx$ | 36 | $\int (\frac{\sin x}{8} + \frac{3}{\cos^2 x})dx$ |
| 37 | $\int \frac{ctgx}{\sin x\cdot\cos x}dx$ | 38 | $\int \frac{2tg2x}{\sin x\cdot\cos x}dx$ |
| 39 | $\int \frac{tgx}{\sin 2x}dx$ | 40 | $\int \frac{tgx}{\cos 2x}dx$ |

**3.** Анықталмаған интегралды табыңыз:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $\int e^{x^2}\cdot x\,dx$ | 2 | $\int e^{-x^3}x^2 dx$ | 3 | $\int \frac{e^x dx}{e^x + 8}$ |

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|---|---|---|---|---|---|
| 4 | $\int x\cos 5x^2 dx$ | 5 | $\int e^{\frac{x^2}{2}} x dx$ | 6 | $\int \frac{e^{2x}dx}{e^{3x}+5}$ |
| 7 | $\int e^{x^2+3}\cdot xdx$ | 8 | $\int \frac{xdx}{\sqrt{x^2+9}}$ | 9 | $\int \frac{\ln^7 x dx}{x}$ |
| 10 | $\int x\cdot\sqrt{x^2+1}dx$ | 11 | $\int x\sqrt[3]{x^2+3}dx$ | 12 | $\int \frac{dx}{x\cdot\ln^{10} x}$ |
| 13 | $\int \frac{xdx}{\sqrt{x^2+9}}$ | 14 | $\int \frac{xdx}{\sqrt{x^2+10}}$ | 15 | $\int \frac{xdx}{\sqrt{29+x^2}}$ |
| 16 | $\int \frac{e^x dx}{e^x+5}$ | 17 | $\int \frac{xdx}{\sqrt{12-x^2}}$ | 18 | $\int \frac{e^x dx}{e^x+7}$ |
| 19 | $\int \frac{e^x dx}{e^{2x}-5}$ | 20 | $\int \frac{e^x dx}{e^{2x}+10}$ | 21 | $\int \frac{e^{2x}dx}{e^{5x}+15}$ |
| 22 | $\int \frac{\ln^2 x dx}{x}$ | 23 | $\int \frac{dx}{x\cdot\ln^3 x}$ | 24 | $\int \frac{\ln^7 x dx}{x}$ |
| 25 | $\int \frac{dx}{x\cdot\ln x}$ | 26 | $\int \frac{xdx}{\sqrt{x^2-11}}$ | 27 | $\int \frac{dx}{x\cdot\ln^9 x}$ |
| 28 | $\int \frac{xdx}{\sqrt{19-x^2}}$ | 29 | $\int \frac{xdx}{\sqrt{20-x^2}}$ | 30 | $\int \frac{xdx}{\sqrt{25+x^2}}$ |
| 31 | $\int \frac{xdx}{\sqrt{22+x^2}}$ | 32 | $\int \frac{3xdx}{\sqrt{x^2+9}}$ | 33 | $\int \frac{e^{3x}dx}{e^{8x}-14}$ |
| 34 | $\int \frac{e^{3x}dx}{e^{4x}-10}$ | 35 | $\int \frac{xdx}{\sqrt{30+x^2}}$ | 36 | $\int \frac{dx}{x\cdot\ln^{12} x}$ |
| 37 | $\int \frac{dx}{x\cdot\ln^5 x}$ | 38 | $\int \frac{xdx}{\sqrt{23+x^2}}$ | 39 | $\int \frac{3xdx}{\sqrt{x^2+11}}$ |
| 40 | $\int \frac{xdx}{\sqrt{36+x^2}}$ | | | | |

**4.** Анықталмаған интегралды табыңыз:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $\int \cos^3 x dx;$ | 2 | $\int \sin^3 x dx.$ | 3 | $\int \frac{2\sin 2x}{\cos^3 x}dx$ |
| 4 | $\int \sin^4 x dx;$ | 5 | $\int tg^2 2x dx;.$ | 6 | $\int \cos^4 \frac{x}{2}dx$ |
| 7 | $\int \cos^5 2x dx;$ | 8 | $\int \cos^7 x dx$ | 9 | $\int \sin^4 2x dx$ |

| | | | | | |
|---|---|---|---|---|---|
| 10 | $\int \frac{\cos x}{\sin^5 x} dx$ | 11 | $\int \sin x \cos^4 x dx$ | 12 | $\int \frac{\cos x}{\sin^6 x} dx$ |
| 13 | $\int \frac{\cos x}{\sin^4 x} dx$ | 14 | $\int \frac{\sin 2x}{\cos^3 x} dx$ | 15 | $\int \frac{\cos 2x}{\cos^2 x} dx$ |
| 16 | $\int \frac{\sin 2x}{\sin^3 x} dx$ | 17 | $\int \sin^3 x \cos^2 x dx$ | 18 | $\int \frac{\sin 2x}{\cos^3 x} dx$ |
| 19 | $\int \sin^8 \frac{x}{2} dx$ | 20 | $\int \cos^6 x dx$ | 21 | $\int \cos^8 \frac{x}{2} dx$ |
| 22 | $\int \sin^4 x dx$ | 23 | $\int \sin^4 x \cdot \cos^3 x dx$ | 24 | $\int \sin^4 3x dx$ |
| 25 | $\int \frac{\cos x}{\sin^4 x} dx$ | 26 | $\int \frac{dx}{\sin^4 x}$ | 27 | $\int \frac{\cos x}{\sin^4 x} dx$ |
| 28 | $\int \frac{2\cos 2x}{\sin^2 x} dx$ | 29 | $\int \sin^6 2x dx$ | 30 | $\int \frac{\cos^2 x}{\sin^4 x} dx$ |
| 31 | $\int \sin^2 x \cos^3 x dx$ | 32 | $\int \sin^3 x \cdot \cos^4 x dx$ | 33 | $\int \frac{4\cos 2x}{\cos^2 x} dx$ |
| 34 | $\int \frac{dx}{\sin^4 3x}$ | 35 | $\int \frac{\cos 2x}{\sin^2 x} dx$ | 36 | $\int \frac{\cos^2 x}{\sin^4 x} dx$ |
| 37 | $\int \frac{\cos^2 x}{\sin^4 x} dx$ | 38 | $\int \sin^6 x dx$ | 39 | $\int \frac{dx}{\sin^4 5x}$ |
| 40 | $\int \frac{2\cos^2 x}{\sin^4 x} dx$ | | | | |

**5.** Анықталмаған интегралды табу керек:

| | | | | | |
|---|---|---|---|---|---|
| 1 | $\int x \cdot e^{2x} dx$ | 2 | $\int (x-1) \cdot e^{-x} dx$ | 3 | $\int \sqrt{x} \cdot \ln 6x dx$ |
| 4 | $\int \frac{\ln x}{x^5} dx$ | 5 | $\int (2x+5) \cdot e^{\frac{x}{3}} dx$ | 6 | $\int x \cdot e^{4x} dx$ |
| 7 | $\int (x-2) \cdot \cos x dx$ | 8 | $\int x \cdot \ln x dx$ | 9 | $\int x \cdot 3^x dx$ |
| 10 | $\int x \cdot \arcsin x dx$ | 11 | $\int x \cos 8x dx$ | 12 | $\int x \cdot \sin \frac{x}{5} dx$ |
| 13 | $\int x \cdot \cos 3x dx$ | 14 | $\int \sqrt{x} \cdot \ln 8x dx$ | 15 | $\int x \cdot \cos \frac{x}{5} dx$ |
| 16 | $\int x^2 e^{3x} dx$ | 17 | $\int \frac{\ln x}{x^2} dx$ | 18 | $\int x \cdot arcctgx dx$ |
| 19 | $\int x \cdot sin10x dx$ | 20 | $\int \frac{x}{8^x} dx$ | 21 | $\int x \cdot arctgx dx$ |

| | | | | | |
|---|---|---|---|---|---|
| 22 | $\int (x+1)\cdot \cos x dx$ | 23 | $\int \sqrt{x}\cdot \ln 4x dx$ | 24 | $\int \frac{x}{2^x} dx$ |
| 25 | $\int x\cdot \sin 5x dx$ | 26 | $\int x\cdot \cos 3x dx$ | 27 | $\int (x-12)\cdot \cos x dx$ |
| 28 | $\int x\cdot 5^x dx$ | 29 | $\int x\cdot \sin \frac{x}{2} dx$ | 30 | $\int x\cdot \cos \frac{x}{3} dx$ |
| 31 | $\int x\cos 9x dx$ | 32 | $\int x\cdot \sin \frac{x}{6} dx$ | 33 | $\int x\cdot sin5x dx$ |
| 34 | $\int x\cdot sin12x dx$ | 35 | $\int x\cdot \cos \frac{x}{8} dx$ | 36 | $\int x\cdot e^{-3x} dx$ |
| 37 | $\int 2x\cdot \arcsin x dx$ | 38 | $\int 2x\cdot arcctgx dx$ | 39 | $\int x^2 e^{5x} dx$ |
| 40 | $\int \frac{x}{6^x} dx$ | | | | |

**6.** Анықталмаған интегралды табу керек:

| | | | |
|---|---|---|---|
| 1 | $\int \frac{dx}{x^2+8x+5}$ | 2 | $\int \frac{dx}{x^2-2x+2}$ |
| 3 | $\int \frac{dx}{x^2-x+1}$ | 4 | $\int \frac{xdx}{x^2-4x+8}$ |
| 5 | $\int \frac{dx}{\sqrt{x^2-6x+10}}$ | 6 | $\int \frac{(x-3)dx}{x^2-6x+10}$ |
| 7 | $\int \frac{dx}{\sqrt{x^2+6x+11}}$ | 8 | $\int \frac{(2x-3)dx}{x^2-10x+26}$ |
| 9 | $\int \frac{dx}{\sqrt{x^2+3x+1}}$ | 10 | $\int \frac{(x+1)dx}{x^2+2x+10}$ |
| 11 | $\int \frac{dx}{\sqrt{1+2x-x^2}}$ | 12 | $\int \frac{(x-1)dx}{x^2-x+4}$ |
| 13 | $\int \frac{dx}{\sqrt{2-2x-x^2}}$ | 14 | $\int \frac{(x-10)dx}{x^2+12x+35}$ |
| 15 | $\int \frac{xdx}{x^2+2x+5}$ | 16 | $\int \frac{(x+6)dx}{x^2+12x+148}$ |
| 17 | $\int \frac{(x-1)dx}{x^2-2x+5}$ | 18 | $\int \frac{x}{x^2-6x-16} dx$ |
| 19 | $\int \frac{(x-3)dx}{x^2+5x+4}$ | 20 | $\int \frac{x}{x^2+4x+20} dx$ |
| 21 | $\int \frac{dx}{x^2+x+2}$ | 22 | $\int \frac{dx}{x^2-2x+2}$ |

| | | | |
|---|---|---|---|
| 23 | $\int \frac{xdx}{x^2+4x+5}$ | 24 | $\int \frac{xdx}{x^2-4x+8}$ |
| 25 | $\int \frac{(x+2)dx}{x^2+6x+10}$ | 26 | $\int \frac{(x-3)dx}{x^2-6x+10}$ |
| 27 | $\int \frac{(2x+3)dx}{x^2+10x+26}$ | 28 | $\int \frac{(2x-3)dx}{x^2-10x+23}$ |
| 29 | $\int \frac{(x-1)dx}{x^2-2x+5}$ | 30 | $\int \frac{(x+1)dx}{x^2+2x+10}$ |
| 31 | $\int \frac{(x+1)dx}{x^2+x+4}$ | 32 | $\int \frac{(x-1)dx}{x^2-x+4}$ |
| 33 | $\int \frac{(x+6)dx}{x^2-12x+35}$ | 34 | $\int \frac{(x-10)dx}{x^2+12x+25}$ |
| 35 | $\int \frac{(x-1)dx}{x^2-2x+5}$ | 36 | $\int \frac{(x+6)dx}{x^2+12x+14}$ |
| 37 | $\int \frac{x}{x^2+6x-1}dx$ | 38 | $\int \frac{x}{x^2-6x-12}dx$ |
| 39 | $\int \frac{xdx}{x^2-4x+4}$ | 40 | $\int \frac{x}{x^2+4x+20}dx$ |

## 6.2 БӨЖ орындау үлгісі

**1.** Анықталмаған интегралды табыңыз:

а) $$\int \frac{x^2-\sqrt[3]{x}}{5\sqrt[3]{x^2}}dx=\int\left(\frac{x^2}{5\sqrt[3]{x^2}}-\frac{\sqrt[3]{x}}{5\sqrt[3]{x^2}}\right)dx=\frac{1}{5}\int\left(x^{2-\frac{2}{3}}-x^{\frac{1}{3}-\frac{2}{3}}\right)dx=$$

$$=\frac{1}{5}\left[\int x^{\frac{4}{3}}dx-\int x^{-\frac{1}{3}}dx\right]=\frac{1}{5}\cdot\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}-\frac{1}{5}\cdot\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+C=\frac{3}{35}\cdot x^{\frac{7}{3}}-\frac{3}{10}\cdot x^{\frac{2}{3}}+C=$$

$$=\frac{3}{35}\sqrt[3]{x^7}-\frac{3}{10}\sqrt[3]{x^2}+C.$$

ə) $$\int\left(\frac{5}{49+4x^2}-\frac{3}{x^2-25}-\frac{4}{\sqrt[8]{(x-4)^3}}+\frac{2}{\sqrt{1-25x^2}}\right)dx=$$

$$=5\int\frac{dx}{(2x)^2+7^2}-3\int\frac{dx}{x^2-25}-4\int(x-4)^{3/8}dx+2\int\frac{dx}{\sqrt{1-(5x)^2}}=$$

$$=\frac{5}{2\cdot7}arctg\frac{2x}{7}-\frac{3}{10}\ln\left|\frac{x-5}{x+5}\right|-4\cdot\frac{(x-4)^{11/8}}{11/8}+\frac{2}{5}\arcsin(5x)+C=$$

$$=\frac{5}{14}arctg\frac{2x}{7}-\frac{3}{10}\ln\left|\frac{x-5}{x+5}\right|-\frac{32(x-4)^{11/8}}{11}+\frac{2}{5}\arcsin(5x)+C.$$

**2.** Анықталмаған интегралды табыңыз:

а) $\int (ctg\frac{x}{5}+tg5x)dx = 5\ln\left|\sin\frac{x}{5}\right| - \frac{1}{5}\ln|\cos 5x| + C$.

ә) $\int 2^{3x+5}dx = \frac{1}{3}\int 2^{3x+5}d(3x+5) = \frac{2^{3x+5}}{3\ln 2} + C.$

**3.** Анықталмаған интегралды табыңыз:

$$\int \frac{x^2 dx}{(5+2x^3)^2} = \frac{1}{6}\int \frac{d(5+2x^3)}{(5+2x^3)^2} = -\frac{1}{6(5+2x^3)} + C.$$

**4.** Анықталмаған интегралды табыңыз:

$$\int \frac{\cos x}{\sqrt{\sin^3 x}}dx = \left|\sin x = t, \cos x dx = dt\right| = \int \frac{dt}{\sqrt{t^3}} = -\frac{2}{\sqrt{t}} + C = -\frac{2}{\sqrt{\sin x}} + C$$

**5.** Анықталмаған интегралды табыңыз:

$$\int x\ln x dx = \begin{vmatrix} u = \ln x & du = \frac{dx}{x} \\ dv = xdx & v = \frac{x^2}{2} \end{vmatrix} = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}dx = \frac{x^2\ln x}{2} - \frac{1}{2}\int x dx =$$

$$= \frac{x^2\ln x}{2} - \frac{x^2}{4} + C = \frac{x^2}{4}(2\ln x - 1) + C.$$

**6.** Анықталмаған интегралды табыңыз:

а) $$\int \frac{(x-1)dx}{x^2-x-1} = \int \frac{\frac{1}{2}(2x-1)-\frac{1}{2}}{x^2-x-1}dx = \frac{1}{2}\int \frac{(2x-1)dx}{x^2-x-1} - \frac{1}{2}\int \frac{d\left(x-\frac{1}{2}\right)}{\left(x-\frac{1}{2}\right)^2 - \frac{5}{4}} =$$

$$= \frac{1}{2}\ln\left|x^2-x-1\right| - \frac{1}{2\sqrt{5}}\ln\left|\frac{x-\frac{1}{2}-\frac{\sqrt{5}}{2}}{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}\right| + C = \frac{1}{2}\ln\left|x^2-x-1\right| - \frac{1}{2\sqrt{5}}\ln\left|\frac{2x-1-\sqrt{5}}{2x-1+\sqrt{5}}\right| + C.$$

ә) $\int \frac{2x^2-7x-33}{(x-3)(x^2+9)}dx = \int\left(\frac{A}{x-3} + \frac{Bx+D}{x^2+9}\right)dx$

Ортақ бөлімге келтіре отырып, алымдарын теңестіреміз:

$2x^2-7x-33 = A(x^2+9) + (Bx+D)(x-3$

$x=3$ $\qquad 18-21-33 = A(9+9)$

$18A = -36$ $\qquad A = -2,$

$x = 0$ $\quad -33 = 9A - 3D$

$\quad 3D = 9A + 33 = -18 + 33 = 15 \quad D = 5,$

$x = -3$ $\quad 18 + 21 - 33 = 18A + 18B - 6D$

$\quad 18B = 6 - 18A + 6D = 6 + 36 + 30 = 72$

$\quad 18B = 72 \quad B = 4.$

Сонымен,

$$\int \frac{2x^2 - 7x - 33}{(x-3)(x^2+9)}dx = -2\int \frac{dx}{x-3} + \int \frac{4x+5}{x^2+9}dx = -2\ln|x-3| + \int \frac{4xdx}{x^2+9} + \int \frac{dx}{x^2+9} =$$

$$= -2\ln|x-3| + 2\ln|x^2+9| + \frac{5}{3}arctg\frac{x}{3} + C = 2\ln\left|\frac{x^2+9}{x-3}\right| + \frac{5}{3}arctg\frac{x}{3} + C$$